ASTR 340: Origin of the Universe

Prof. Benedikt Diemer

Lecture 4 • Physical laws for the Universe

09/09/2021

Homework #1

- Available on Canvas after the lecture
- Due **Tuesday 9/21** at midnight
- Questions in pdf file
- Submit a pdf file (handwritten or digital)
- No late submissions!
- Read introduction
- **Show your work**
  Many answers are googleable
- Come to **office hours!**

ASTR 340 (Fall 2021) • Homework 1

Prof. Benedikt Diemer

Due Tuesday 9/21/2021 • Covers lectures 1 to 4 • Please do not share this document online!

Introduction

The main purpose of this first homework is to familiarize you with the types of units and quantities we will encounter throughout the course. A few introductory remarks:

Scientific notation: It is tempting to just write down the result that your calculator spits out from a given computation. For example, say you're dividing 6.67×10^{10} by 3.26×10^4, which gives 2046012.27. While that may be the correct answer numerically, it isn't the correct way to express it! First, large numbers get hard to read because you need to count digits to know whether you're talking millions, tens of millions, and so on. Second, by giving 9 significant digits, you are implying that you know the answer to that accuracy — which you don't because the input quantities had only three significant digits. Instead, please express numbers in scientific notation with a number of digits that (roughly) matches the inputs to the calculation, e.g., 2.05×10^6 for the example above. This style will also make your grader happy! In general, I will give constants with three significant digits. If higher accuracy than three digits is needed, I will specifically say so in the problem.

Units: One of the main challenges is to keep track of units. In astronomy, we most commonly use the centimeter-gram-second (cgs) system. In cgs units, the speed of light is 3.00×10^{10} cm/s, an "astronomical unit" (AU = Earth-Sun distance) is 1.50×10^{13} cm, and the gravitational constant is $G = 6.67 \times 10^{-8}\ \mathrm{cm^3 g^{-1} s^{-2}}$. The most common mass unit in this course will be the solar mass, 2.00×10^{33} g. Furthermore, an arc-minute is 1/60 of a degree, and an arc-second is 1/60 of an arc-minute or 1/3600 of a degree.

Side note: you might wonder why astronomers use cgs when physicists generally use meter-kilogram-second (mks). Given the large distances in astronomy, would meter not make more sense than cm? The point is that the distances are so large that it really doesn't matter; the choice of cgs over mks is historical.

Question 1: Angular sizes [20 points]

Part a) The moon measures about 32 arc-minutes in the sky (close to half a degree). The distance to the moon varies somewhat with time, but let's take 3.73×10^{10} cm. What is the radius of the moon, R_{moon}, in km? [6 points]

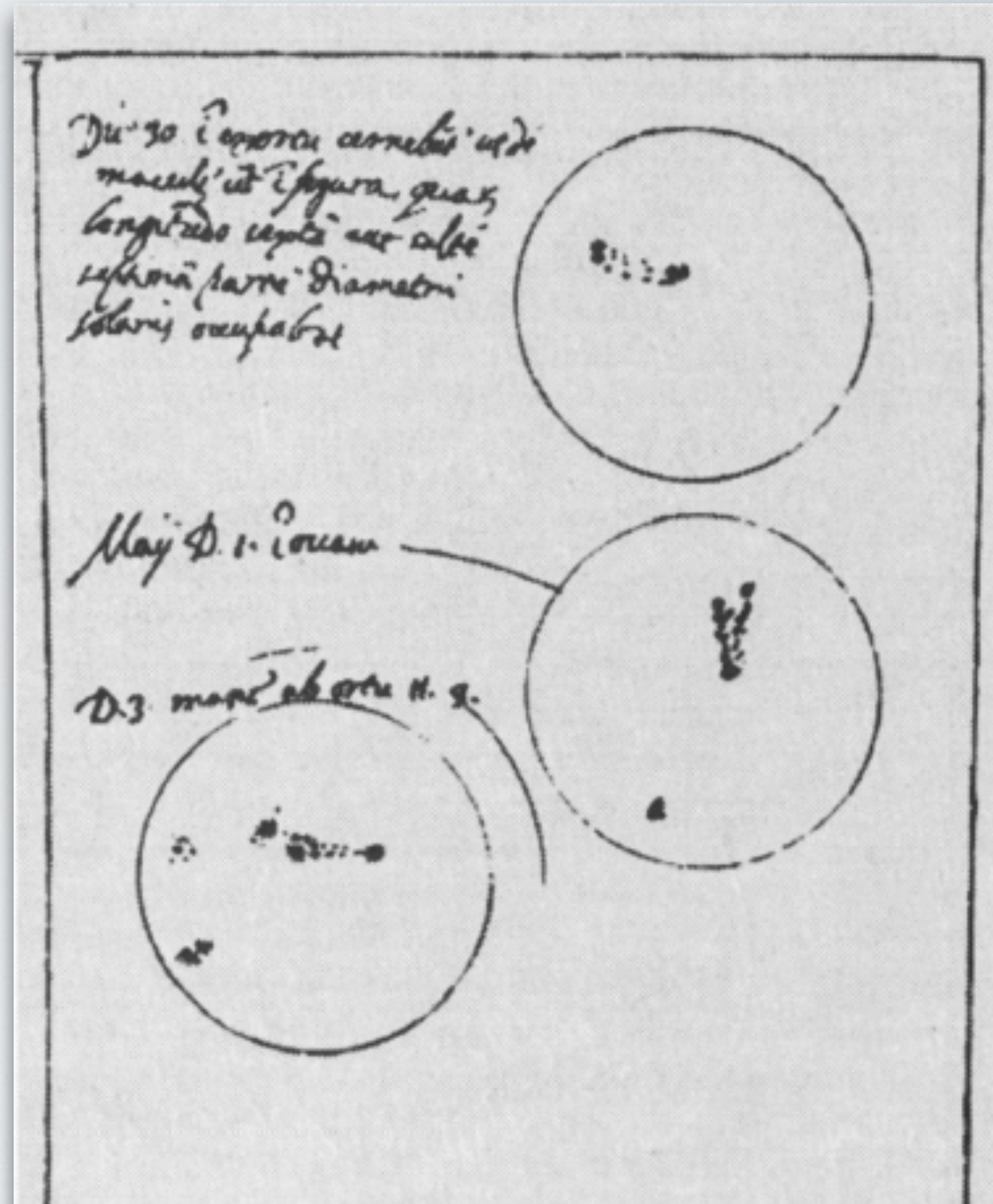
Part b) Coincidentally, the moon and Sun share almost exactly the same angular diameter on the sky, but the Sun is about 400 times more distant. Given your result for the radius of the moon, what is the radius of the Sun, $R_\odot$? [2 points]

Part c) Derive the length unit of a parsec (pc), defined as the distance of a star if its parallax (=angular shift) is one arc-second. Start by drawing a diagram of the motion of the Earth around the Sun and the corresponding angular shift in the direction of a star. What is a pc in AU and in cm? [12 points]

Weekly schedule

Time	Monday	Tuesday	Wednesday	Thursday	Friday
11:00-12:00	TA office hours			Office hours	
12:00-12:30					
12:30-1:45		Lecture		Lecture	
1:45–3:00					
3:00-4:00			Office hours		
4:00-11:59					
11:59			Tue quiz due		Thu quiz due

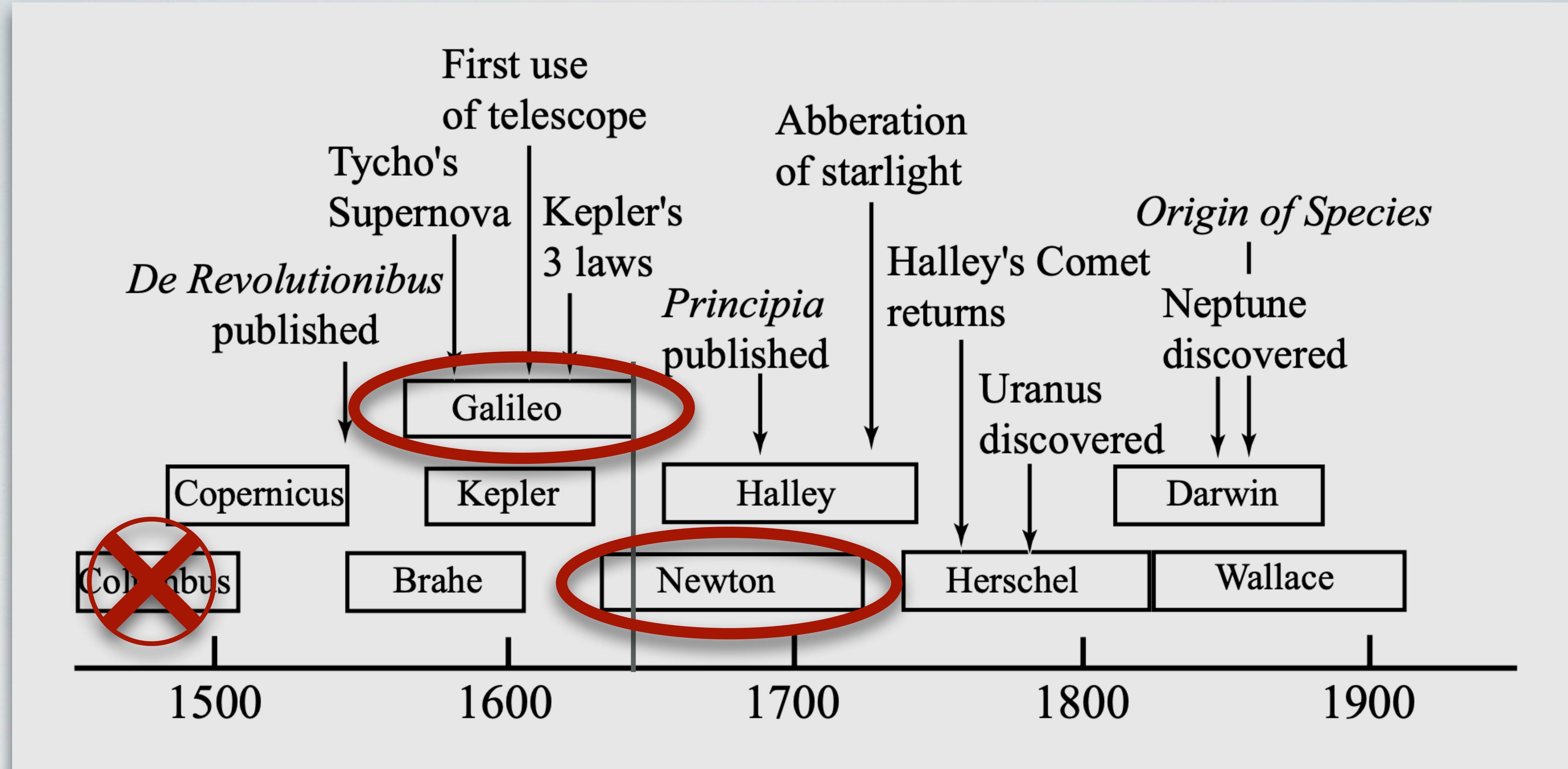
Galileo's observations: Sunspots



Free Parameters (from group discussion)

- Clarification: question was about free parameters **in model of solar system**, not underlying theory
  - Newtonian theory has only one free parameter (G) but still cannot predict the details of the solar system
- One element of the model is **not the same as one free parameter**
  - One sphere means multiple free parameters (radius, speed, offset…)
  - One point in space (e.g., equant) can be a **vector**, meaning multiple free parameters (one for each direction)
- Scientific method: **fewer free parameters** means more…
  - **Accurate?** Not in general
  - **Consistent?** Not necessarily
  - **Falsifiable?** Possibly
  - **Predictive?** Yes!

Today



Hawley & Holcomb, Foundations of Modern Cosmology

Part 1: Galilean physics

Galileo's physics

- After 1633 trial, Galileo returned to work on **mechanics**
- From experiments with inclined planes, concluded that distance *d* traveled under **uniform acceleration** *a* is $d = at^2$

Image: Museo Galilei

Galileo’s physics

- Used “thought experiments” to conclude that all bodies, regardless of mass, fall at the same rate in a vacuum
  - Now known as “**equivalence principle**”
- Realized full **principle of inertia**
  - body at rest remains at rest
  - body in motion remains in motion (force not required, contrary to Aristotle)
- Realized principle of **relative motion** (“Galilean invariance”)
  - If everything is moving together at constant velocity, there can be **no apparent difference** from case when everything is at rest

Participation: Galilean invariance 1 & 2

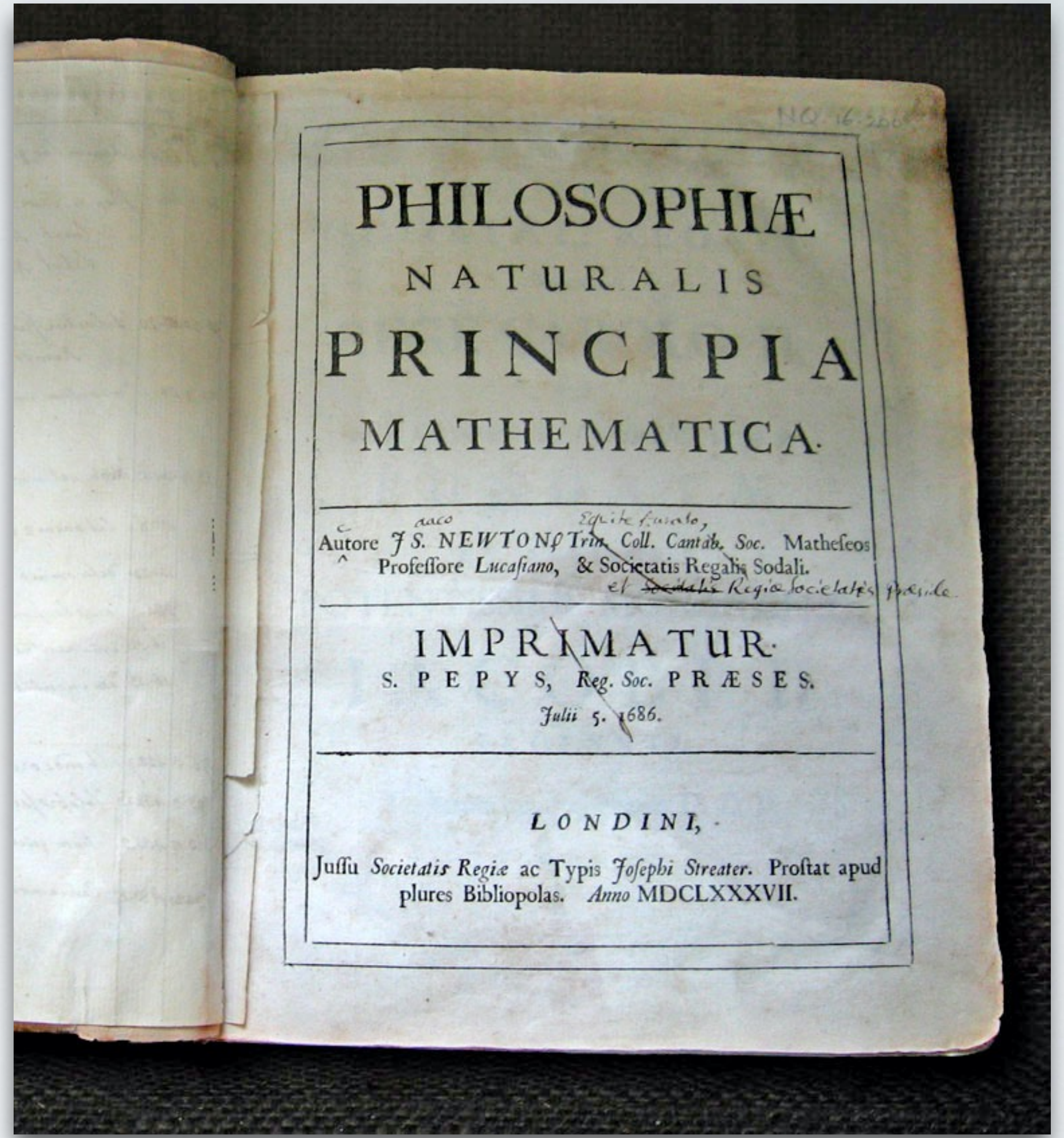
Respond to the poll on TurningPoint

Session ID: diemer

30 seconds

Part 2: Newton’s history

Isaac Newton (1642-1727)



Painting: Sir Godfrey Keller (1689)

Isaac Newton (1642-1727)

- Attended **Cambridge University** (Trinity College)
  - Intended to study law, but read Kepler, Galileo, Descartes
  - Began to study mathematics in 1663
  - **Pandemic!** Cambridge closed due to plague 1665-1667; Newton quarantined and worked out foundations of **calculus**
- Became **professor of mathematics** in 1669 (age 27!)

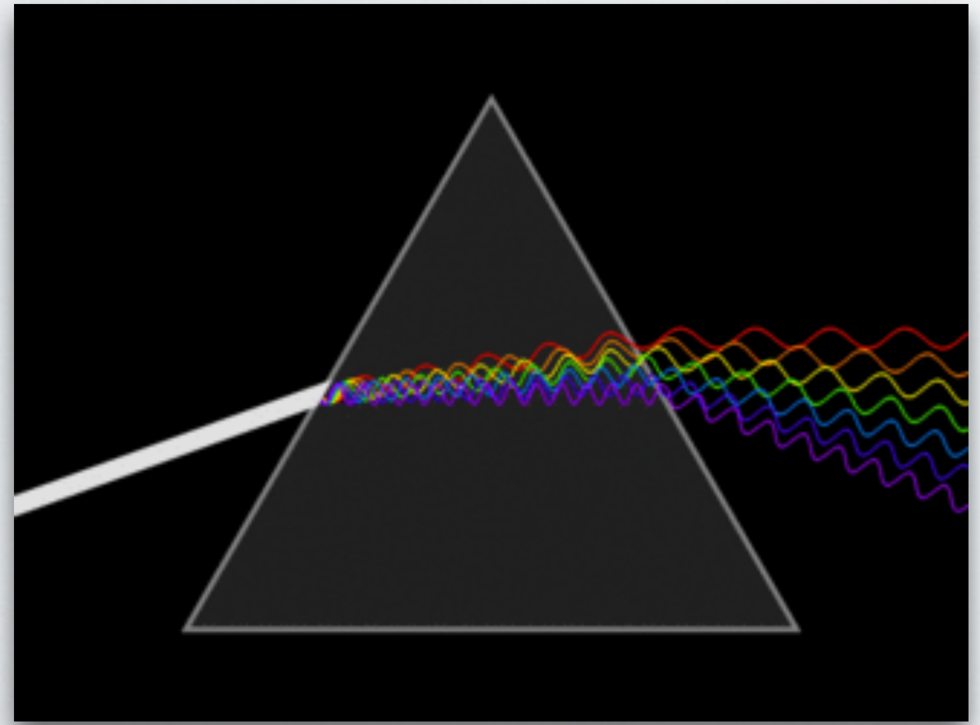
$$f(x) = \int_0^x f'(t)\, dt$$

(independently discovered

by Leibniz)

Isaac Newton (1642-1727)

- Worked in **optics**, publishing "Opticks" (1704)
    - Realized (contrary to Aristotle) that white light is not a single entity but composed of many colors
    - Invented **reflecting telescope**
    - Analyzed diffraction phenomenon: **light is a wave**



Images: Apic/Getty Images

Isaac Newton (1642-1727)

- In 1687, published Philosophiae naturalis principia mathematica, or **“Principia”**
  - Publication was prompted (and paid for) by Halley
  - Generalized Sun’s gravity law to universe law of gravitation: **all matter attracts** all other matter with a force proportional to the product of their masses and inversely proportional to the square of the distance between them
  - Partly in response to Hooke’s claim that he could prove gravity obeys inverse-square law
  - Many other applications, including tides, precession, etc.
  - Laid out general physics of **mechanics**: the laws of motion
  - Showed that Kepler’s laws of **ellipses** follows from more fundamental laws
- The Principia is recognized as the greatest scientific book ever written
- Retired from research in 1693, becoming active in politics and government

Participation: Newton's contributions

Respond to the poll on TurningPoint

Session ID: diemer

30 seconds

Part 3: Newton's Laws

Newton's 1st law

If no force acts upon a body, its velocity remains constant.

- No natural state of "**being at rest**" (Aristotle)
- Includes special case $v = 0$ (a body remains at rest if $F = 0$)

Participation: Newton's 1st law

Respond to the poll on TurningPoint

Session ID: diemer

30 seconds

Newton's 2nd law

If a body of mass *m* is acted upon by a force *F*, then its acceleration *a* is given by *F* = *ma*

- Defines "**inertial mass**" as the degree by which a body resists being accelerated
- Since momentum **p = mv** and acceleration a = rate of change in v,
  ma = rate of change in (mv)
  - This form is more general, since it includes case when **mass is changing**
  - **Momentum is conserved if F = 0**

Force = rate of change of momentum $$F = \frac{d(mv)}{dt}$$

Newton's 3rd law

If body A exerts force $F_{A\to B} = f$ on body B, then body B exerts a force $F_{B\to A} = -f$ on body A.

- Often phrased in terms of "**equal**" (in magnitude) and "**opposite**" (in direction) forces
- **Total force on a closed system is 0**, i.e.
  $F_{tot} = F_{A\to B} + F_{B\to A} = f + (-f) = 0$
- Combined with 2nd law, this implies that the total momentum of a closed system is conserved if there are no external forces, i.e.
  $F_{tot} = 0 \Rightarrow$ rate of change of p_{tot} is $0 \Rightarrow$ **p_{tot} = constant**

Newton's 3rd law

Professor Goddard … does not know the relation between action and reaction and of the need to have something better than a vacuum against which to react… he seems to lack the basic knowledge ladled out daily in high schools.

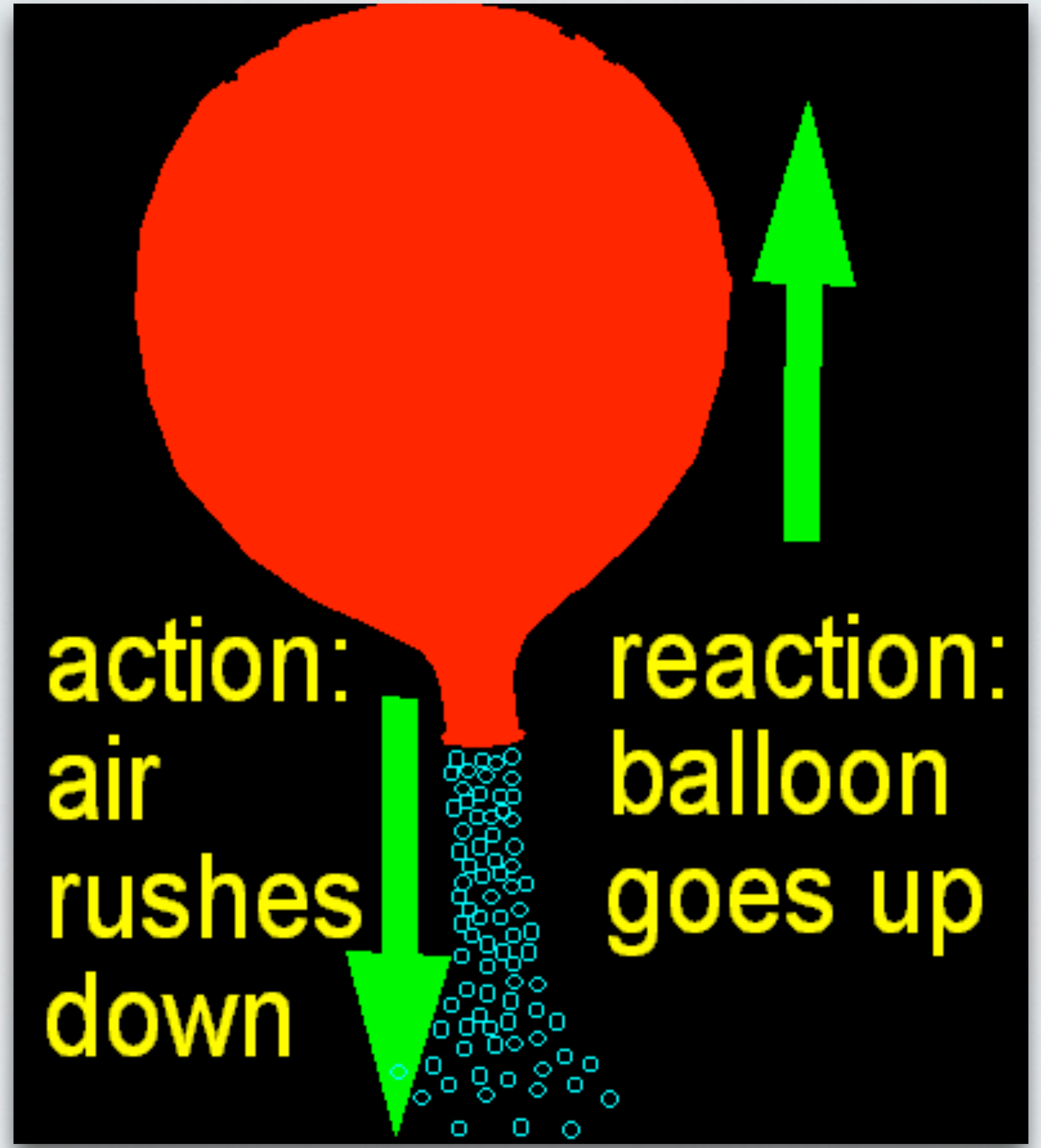
New York Times editorial on Robert Goddard's proposal that rockets could reach Moon

SUNDAY, THREE CENTS. DAILY, ONE CENT.

"All the News That's Fit to Print."

The New York Times.

Newton's 3rd law



Newton’s 3rd law

Newton’s 3rd law

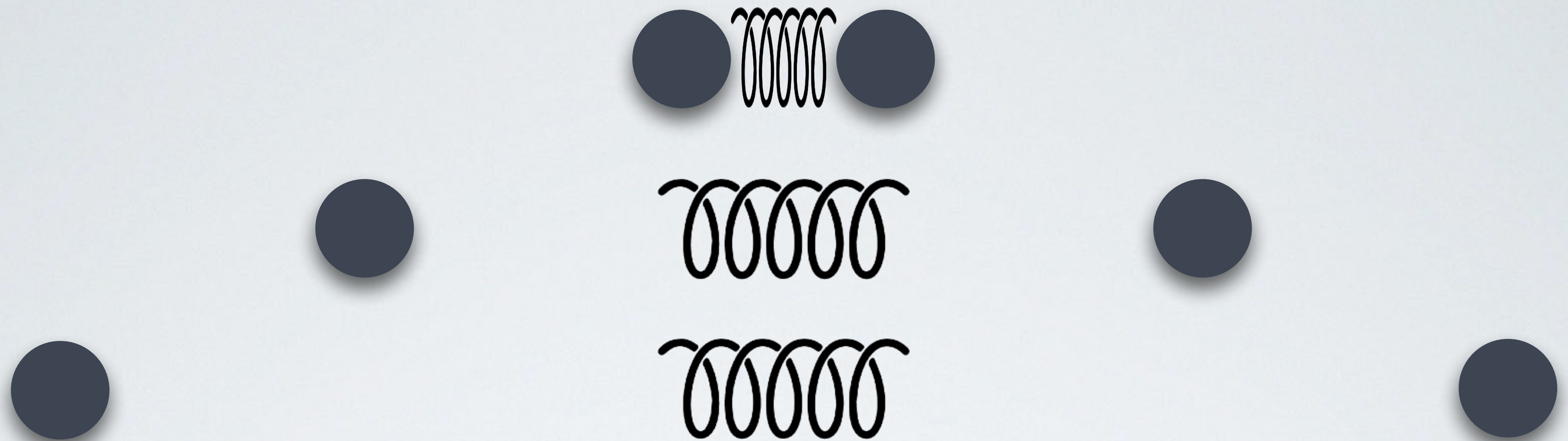
A Correction

On Jan. 13, 1920, “Topics of The Times,” an editorial-page feature of The New York Times, dismissed the notion that a rocket could function in a vacuum and commented on the ideas of Robert H. Goddard, the rocket pioneer, as follows:

“That Professor Goddard, with his ‘chair’ in Clark College and the countenancing of the Smithsonian Institution, does not know the relation of action to reaction, and of the need to have something better than a vacuum against which to react—to say that would be absurd. Of course he only seems to lack the knowledge ladled out daily in high schools.”

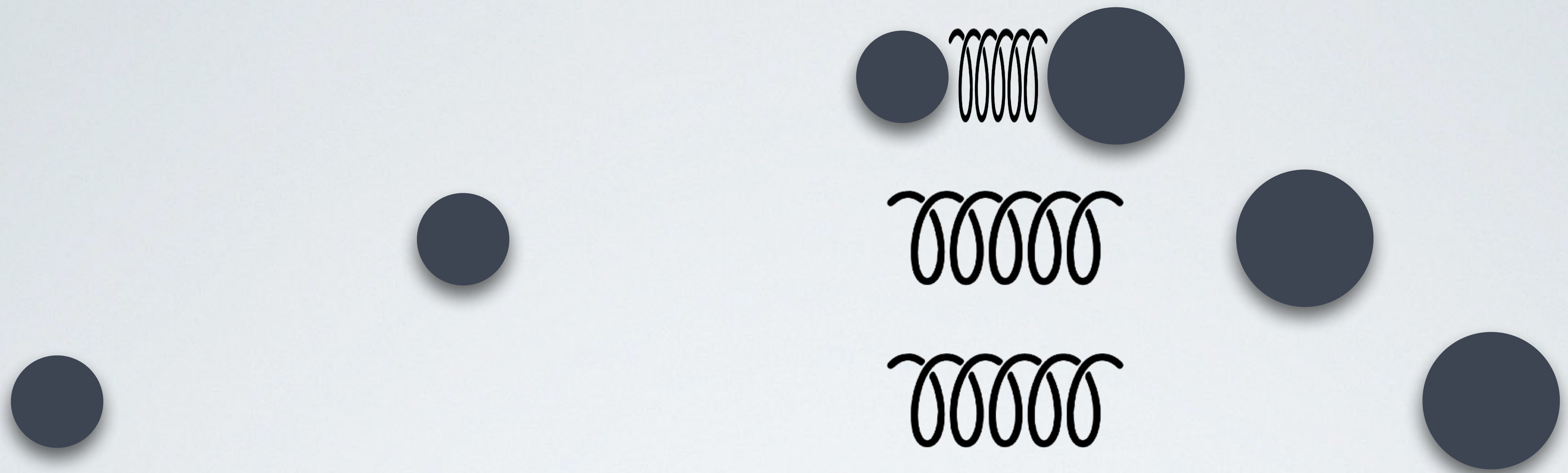
Further investigation and experimentation have confirmed the findings of Isaac Newton in the 17th Century and it is now definitely established that a rocket can function in a vacuum as well as in an atmosphere. The Times regrets the error.

Combining Newton's laws



- Two equal masses M at rest; initial momentum is p = 0
- Masses are suddenly pushed apart (**accelerated**) by a spring (**2nd** law)
- Will move apart with the same speed V in **opposite** directions (**3rd** law)
  - Total momentum is p = MV-MV = 0
  - Total momentum is unchanged
- Will **keep moving** at constant speed (**1st** law)

Combining Newton's laws



- Acceleration depends on mass, as in a = F / m
- If one mass is larger, it will be accelerated less

Newton's law of gravity

Particles with masses m_1 and m_2 at a distance r will attract each other with a force F given by

$$F = \frac{Gm_1 m_2}{r^2}$$

- G is the gravitational constant, $G = 6.67 \times 10^{-8} \frac{\mathrm{cm}^3}{g\ s^2}$
- Attraction is universal: **every particle** in the Universe attracts every other particle!
- Gravity often **dominates in astronomical settings**

Participation: Discussion #4

Discuss Newton's laws of mechanics and gravity. Find an example where each law is apparent on Earth, and one example where it is not.

5 minutes

General physics problem solving

b. A spaceship is floating in open space (feeling no gravity). Using Newton's 2nd law, explain why the momentum of the ship is not changing. *[6 points]*

c. We now accelerate the spaceship, which weighs 10 tons, by expelling 200kg of rocket fuel at a speed of 400 m/s. How fast is the spaceship traveling after the acceleration has finished? *[6 points]*

Stage 1: Understand physics

- total momentum is not changing
- total momentum is zero
- solve by momentum conservation

Stage 2: Translate to math

Momentum of fuel + momentum of ship = 0

$$m_\mathrm{f} v_\mathrm{f} + m_\mathrm{s} v_\mathrm{s} = 0$$

Stage 3: Solve math

$$\implies m_\mathrm{s} v_\mathrm{s} = -m_\mathrm{f} v_\mathrm{f}$$

$$\implies v_\mathrm{s} = -\frac{m_\mathrm{f} v_\mathrm{f}}{m_\mathrm{s}} = -\frac{200\mathrm{kg}\ 400\mathrm{m/s}}{10{,}000\mathrm{kg}} = -8\mathrm{m/s}$$

Stage 4: Interpret solution

Spaceship is moving in opposite direction at 8m/s

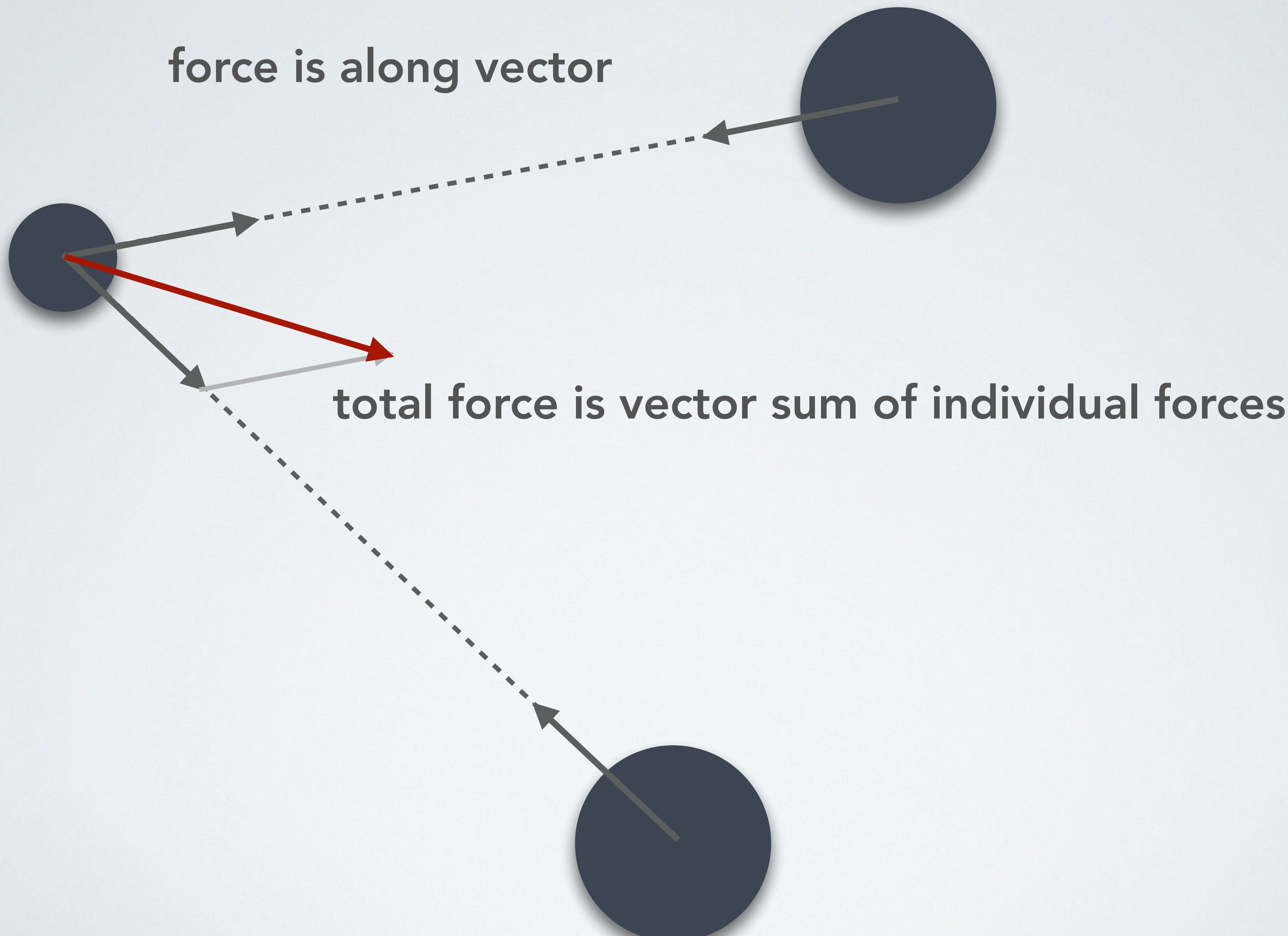
Gravity on Earth

$$m_1 = M_{\text{earth}}$$

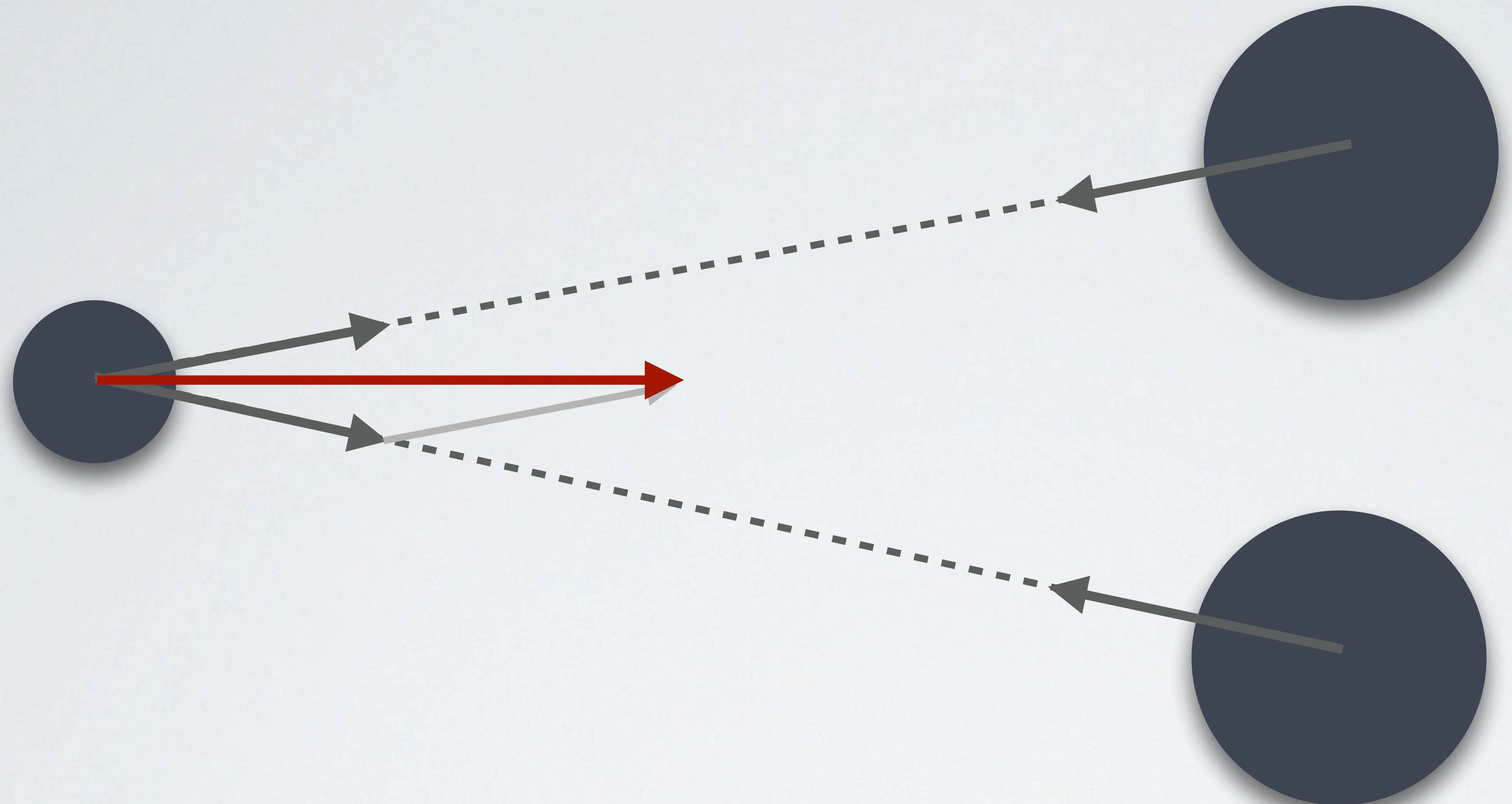
$$r = R_{\text{earth}}$$

$$\implies a = \frac{F}{m_2} = \frac{G\ M_{\text{earth}}}{R_{\text{earth}}^2} \equiv g = 9.8 \frac{m}{s^2}$$

Newton's law of gravity

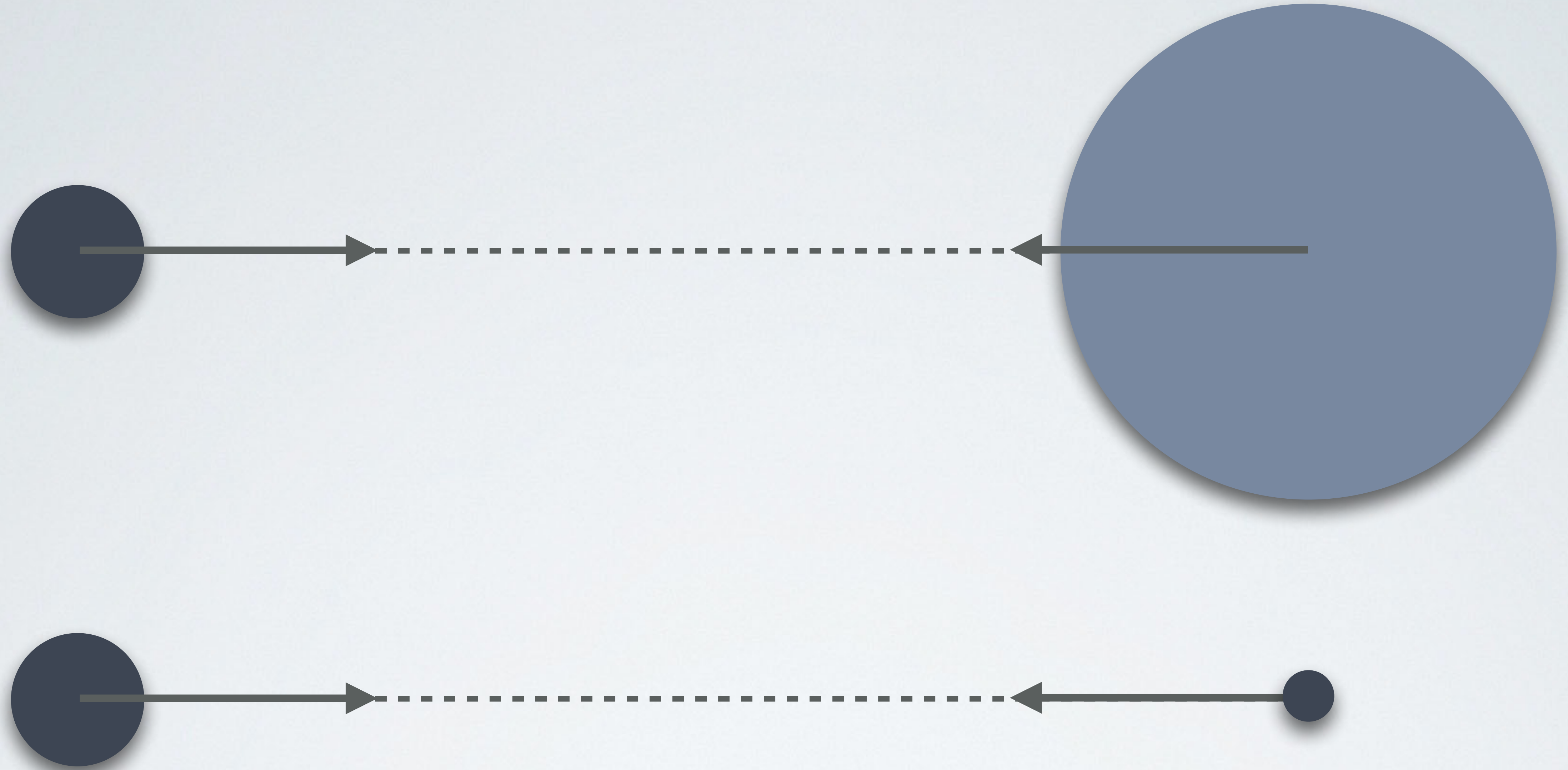


Newton's law of gravity



changing positions changes force vectors

Newton's law of gravity

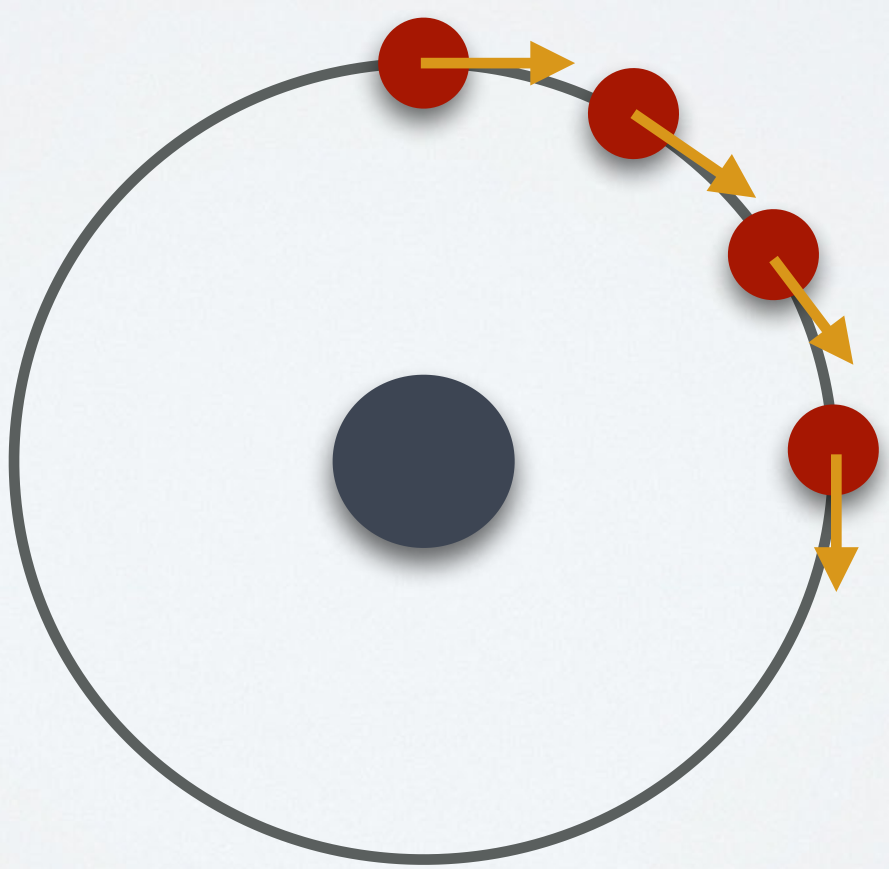


Spherically symmetric object has same gravity as point mass!

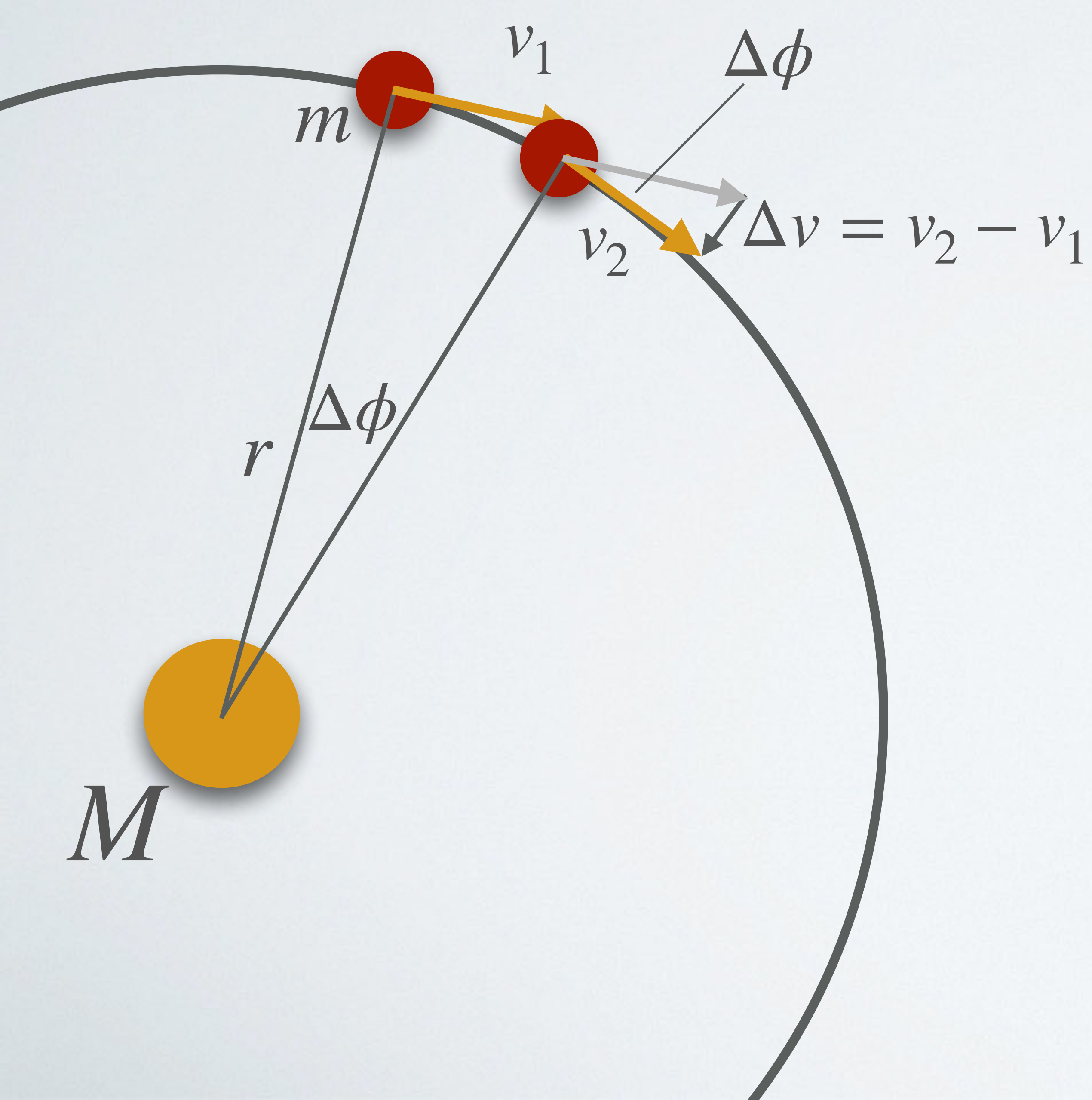
Part 4: From Newton to Kepler

Acceleration in a circular trajectory

- Newton arrived at the theory of gravity by imagining that the **same force** causes an apple to fall towards the Earth as the Moon to orbit Earth
- In what sense is a body in orbit "**falling**"?
- Any change in direction requires **acceleration**
- Acceleration must be towards **center**



Acceleration in a circular trajectory



$$a = \frac{\Delta v}{\Delta t}$$

for small $\Delta\phi$, $\Delta v \approx v\Delta\phi$

$$\implies a = \frac{v\Delta\phi}{\Delta t}$$

orbital time $P = \dfrac{2\pi r}{v}$

$$\frac{\Delta\phi}{2\pi} = \frac{\Delta t}{P} \implies \Delta\phi = \frac{v\Delta t}{r}$$

$$\implies a = \frac{v\Delta\phi}{\Delta t} = \frac{v^2}{r}$$

$$F_{\text{centripetal}} = \frac{mv^2}{r}$$

Kepler’s laws revisited

- **Kepler’s 1st law**
  - Assume planets are gravitationally attracted to Sun
  - Can be derived from Newton’s laws, but the full proof is a bit complicated and involves calculus
  - $1/r^2$ law is exactly what’s needed to create an ellipse
- **Kepler’s 2nd law**
  - Because force is always directed towards Sun
  - Equal areas per time ⇔ conservation of angular momentum
  - Kepler’s 2nd law would be true for any “central” force, not just $1/r^2$

Kepler's 3rd law

gravity = centripetal force

$$F = ma = \frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$P = \frac{2\pi r}{v}$$

$$\frac{GM}{r} = \frac{4\pi^2 r^2}{P^2} \rightarrow P^2 = \frac{4\pi^2}{GM} r^3$$

Participation: 3-body problem

Respond to the poll on TurningPoint

Session ID: diemer

30 seconds

The 3-body problem

Movie: Carl Rodriguez

Newton in perspective

Theory of motion and gravity **removed Aristotle's distinction** between the Earth and the Heavens

- The same phenomena happen **there and here** (Galileo)
- They obey the **same set of physical laws** (Newton)
- The Universe is **knowable**

Newton in perspective

- With Newton's laws, it was possible to make **predictions** about orbits of solar system bodies
  - Halley argued that several comet appearances separated by 76 years were actually the same comet, and predicted its recurrence in 1758
- Planet orbits are **not perfectly elliptical** orbits due to gravity of other planets
  - Herschel, in 1781, discovered Uranus; its orbit showed enough variations to predict there must be another as-yet-unknown planet, leading to discovery of Neptune in 1846
- Huge **cultural impact**
  - A Universe describable by precise mathematical laws supports the idea of "rationality" in other arenas (e.g., architecture, government, history, etc.)

Take-aways

- There is only **one set of physical laws** on Earth and in the Universe
- Newton's laws describe **mechanics and gravity** in a very general way

Next time…

We'll talk about:

- Reference frames and fictitious forces

Assignments

- Post-lecture quiz (by tomorrow night)
- Homework #1 (by Tuesday 9/21/21)

Reading:

- H&H Chapter 6 (up to page 173)