ASTR 340: Origin of the Universe

Prof. Benedikt Diemer

Lecture 9 • General Relativity I

09/28/2021

Today

- Light & redshift
- Free-fall
- The strong equivalence principle
- Basics of General Relativity
- Perihelion precession

Part 1: Light & redshift

Participation: Recap #1 & 2

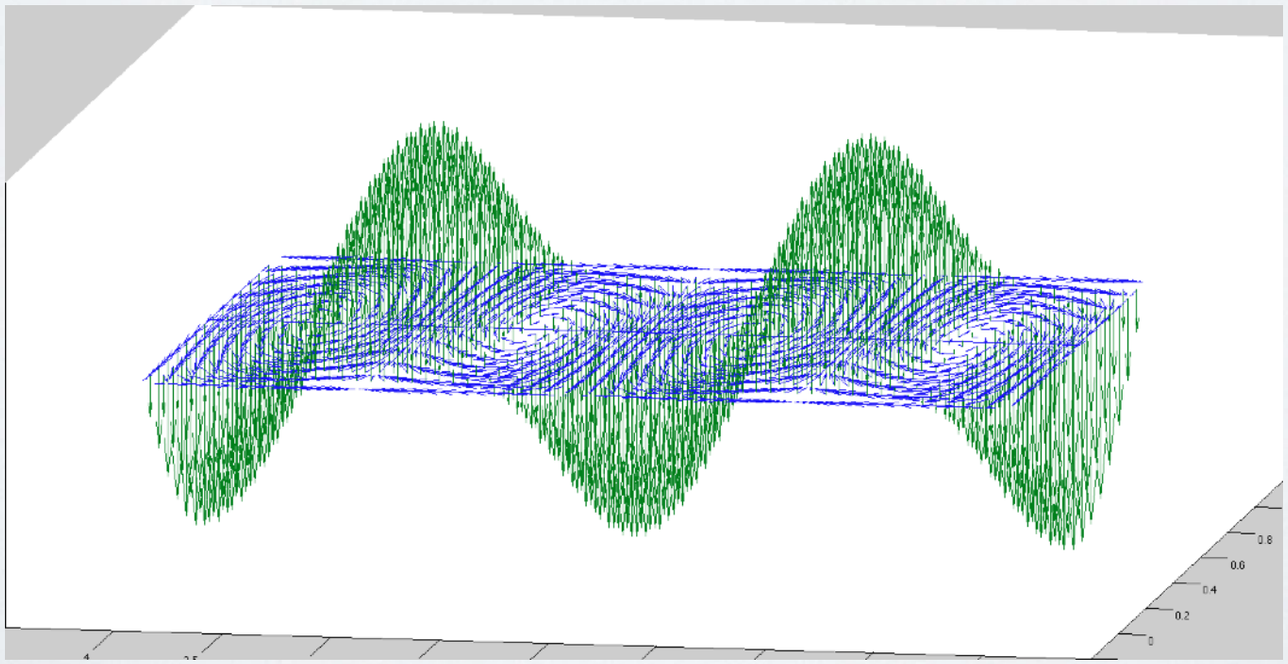
Respond to the poll on TurningPoint

Session ID: diemer

 30 seconds

Light

- Photons (light particles) are massless
- Light has a wave/particle dual nature
- Waves are characterized by
  - Wavelength (λ) = distance between crests
  - Frequency (ν or f) = number of crests passing a given point per second
- **Speed** of a crest is $c=\lambda f$
- **Energy** of a photon is proportional to frequency, **$E=hf$**
  - h = Planck constant = 6.6×10^{-27} cm^2g/s



Participation: Recap #3

Respond to the poll on TurningPoint

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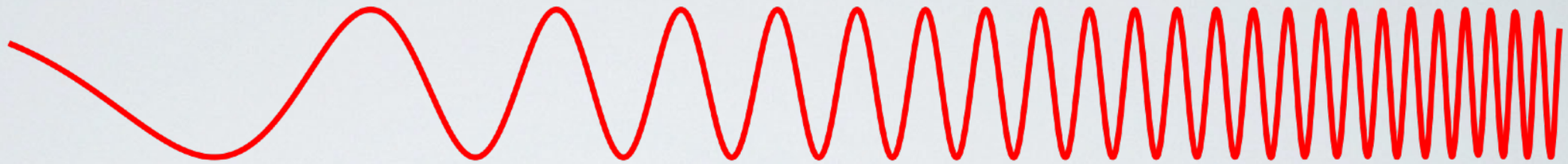
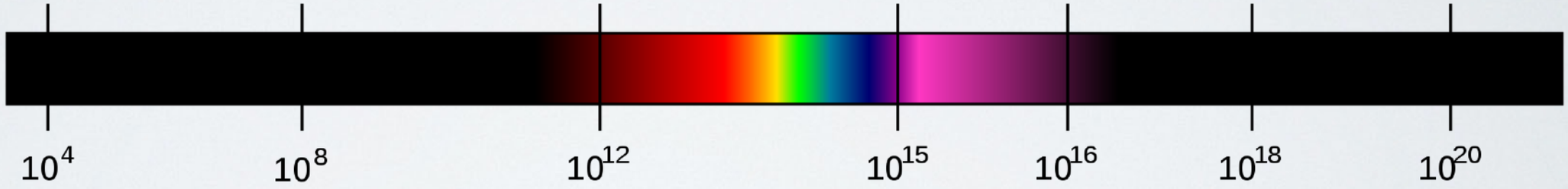
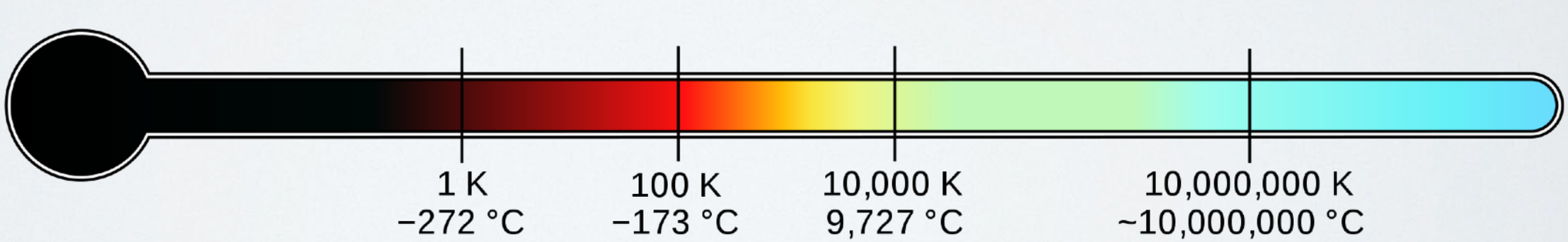
30 seconds

The electromagnetic spectrum



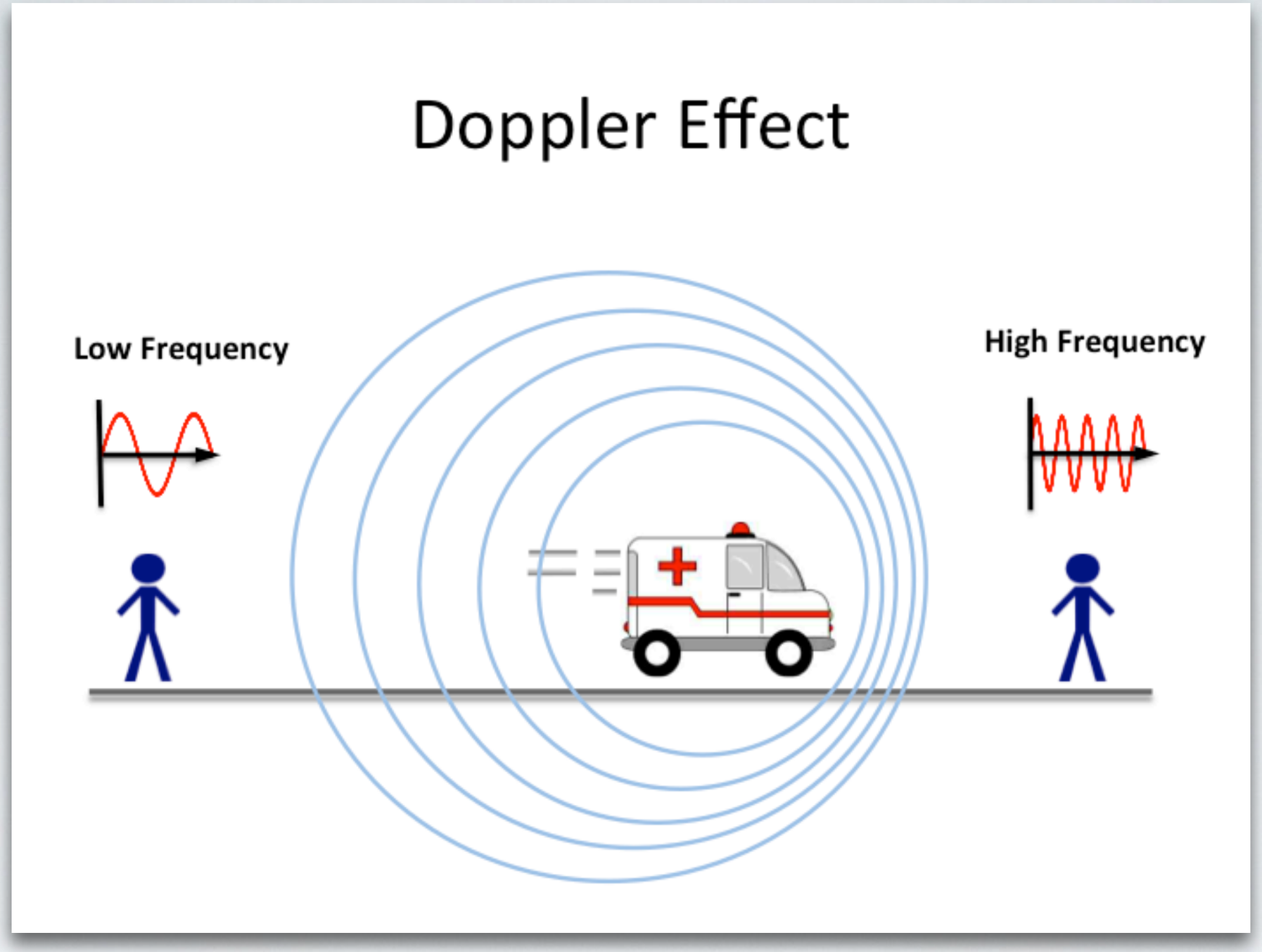
Radiation Type	Radio	Microwave	Infrared	Visible	Ultraviolet	X-ray	Gamma ray
Wavelength (m)	10^{3}	10^{-2}	10^{-5}	0.5×10^{-6}	10^{-8}	10^{-10}	10^{-12}
Approximate Scale of Wavelength	Buildings	Humans	Butterflies	Needle Point	Protozoans	Molecules	Atoms	Atomic Nuclei





low energy — high energy

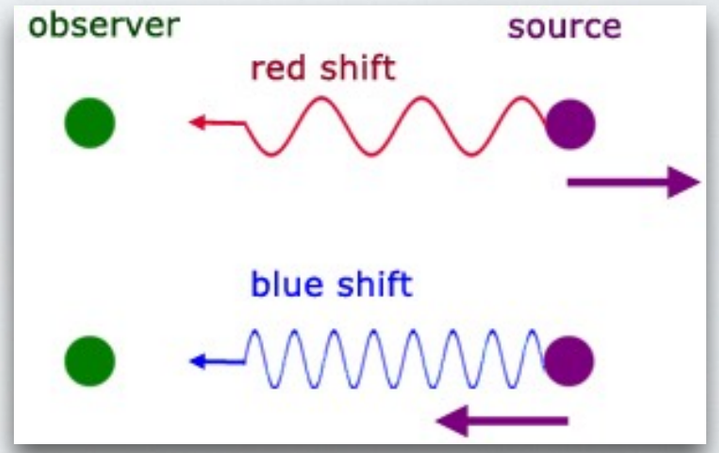
Doppler effect



Relativistic redshift

- Photon energy changes when observer's frame changes
- Energy of a photon is proportional to the frequency of the light wave

$$f_{\mathrm{m}} = f_{\mathrm{p}} \times \gamma \left(1 + \frac{v}{c} \right)$$



- **Moving towards** a light source, the frequency and energy increase by this factor = **blueshift** (bluer, not necessarily blue)
- **Moving away** from a light source, the frequency and energy decrease by this factor = **redshift** (redder, not necessarily red; v is negative)

Part 2: Free-fall & the strong equivalence principle

Recap of Special Relativity

- Einstein's postulates for Special Relativity
  - Laws of physics are the same in any inertial frame of reference
  - Speed of light is the same in any inertial frame of reference
- Strange consequences of Special Relativity
  - Time dilation and length contraction
  - Relativity of simultaneity and ordering of events
  - Equivalence of mass and energy

Conversion from energy to mass

$$E = mc^2$$

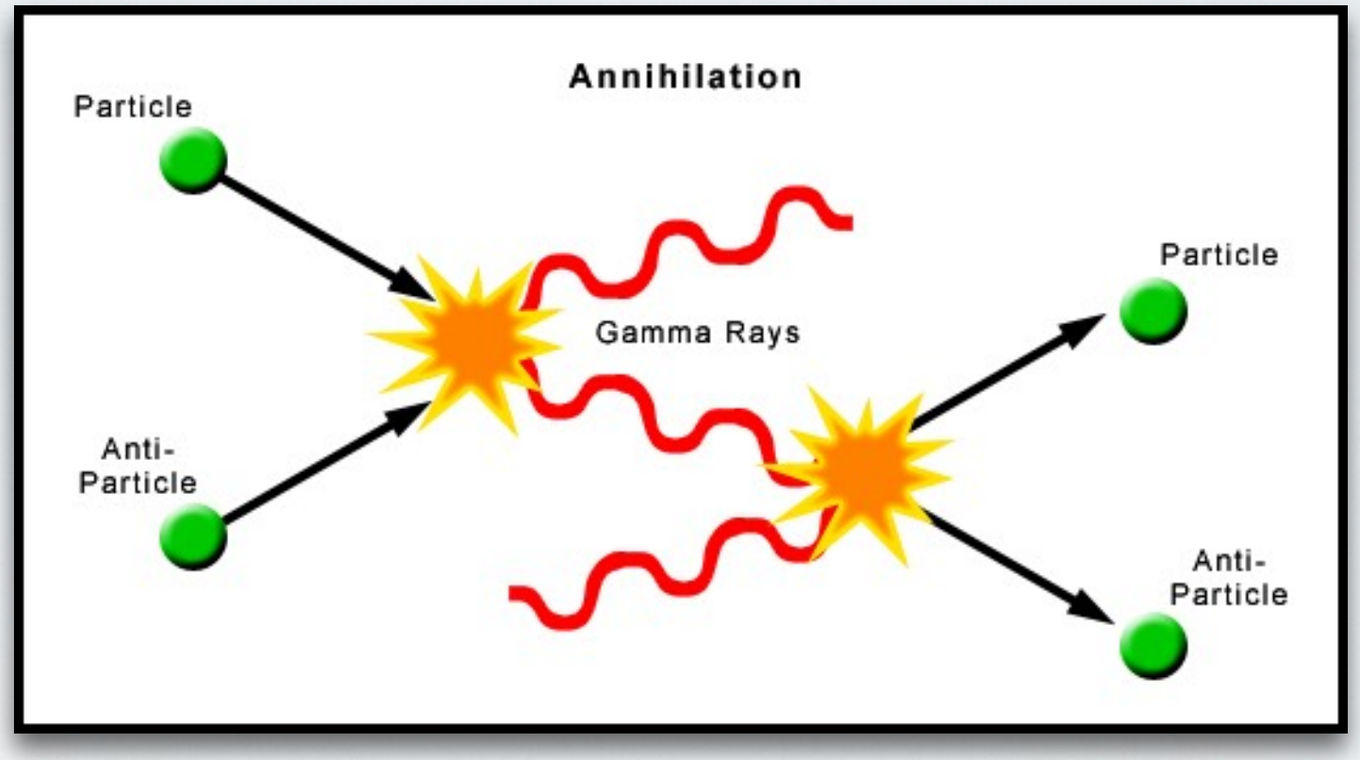


Image: ANU

Participation: Mass-energy

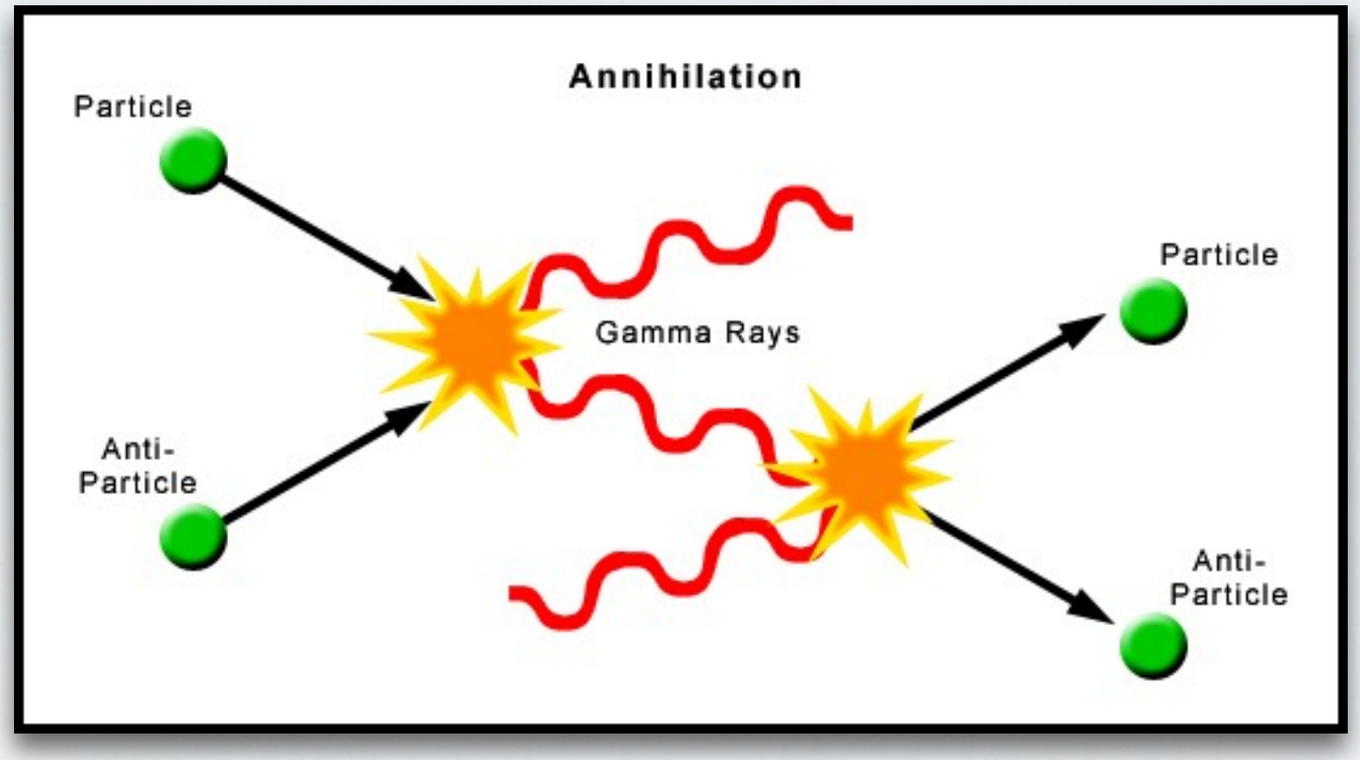
Respond to the poll on TurningPoint

Session ID: diemer

30 seconds

Conversion from energy to mass

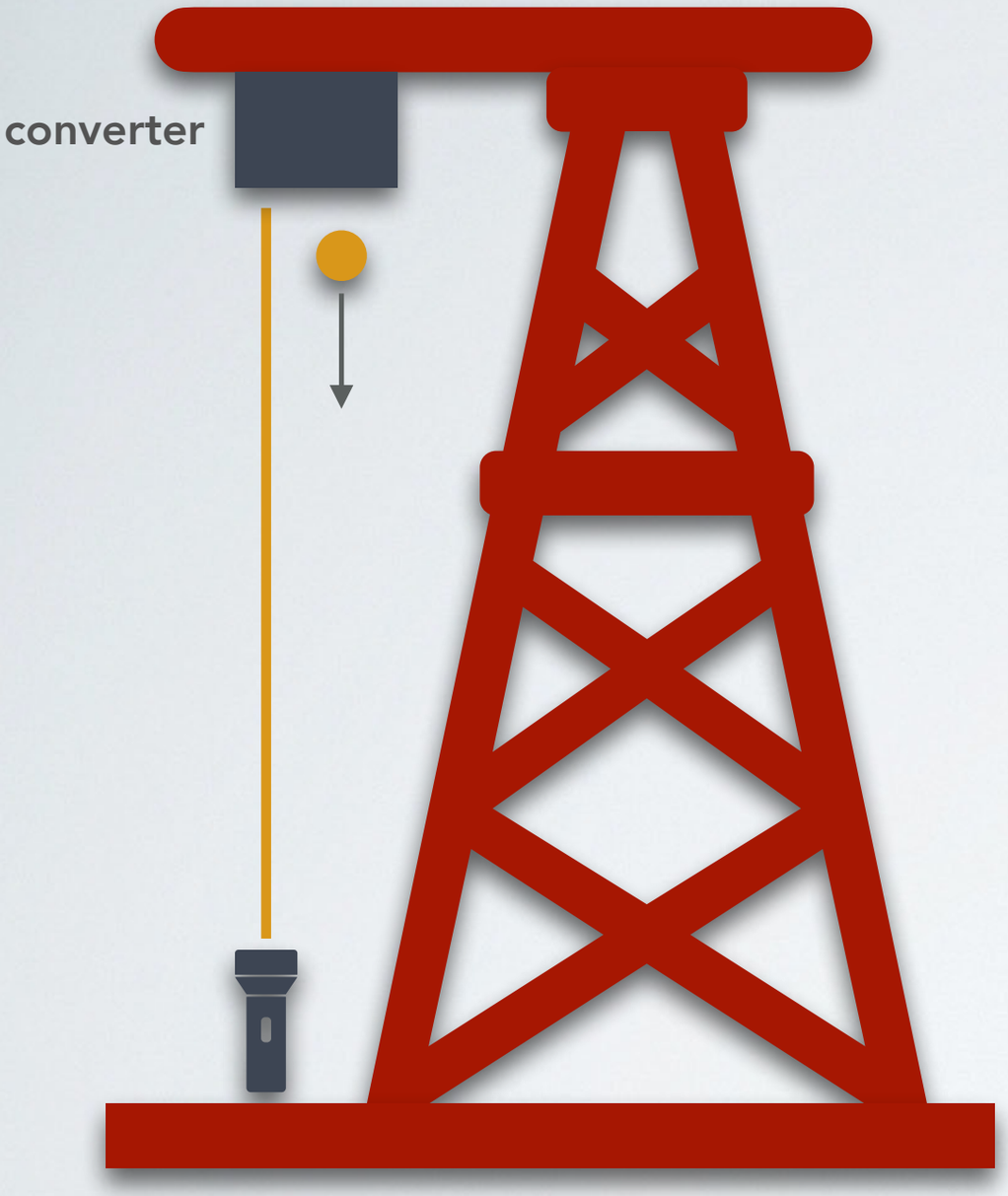
$$E = mc^2$$



E is the total energy, so kinetic energy is also converted.

Image: ANU

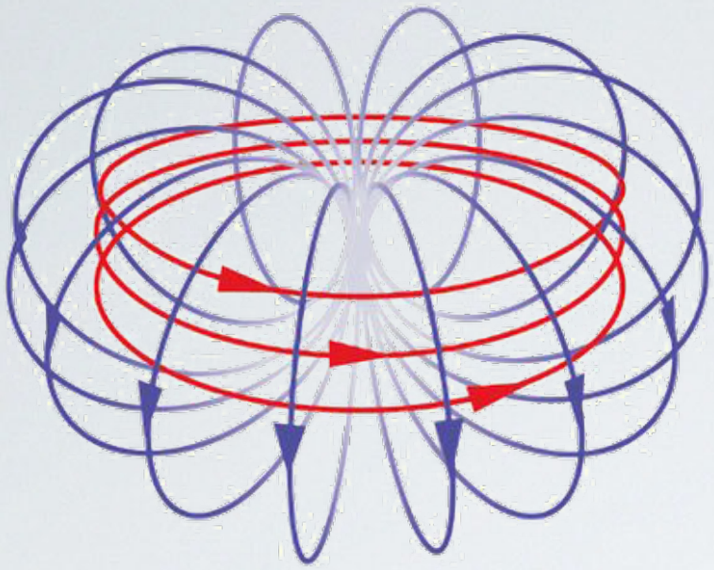
The tower problem

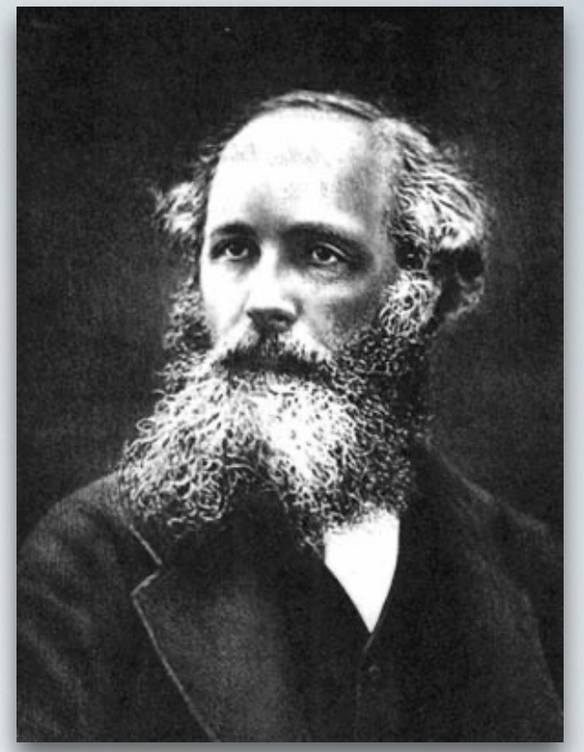


- Thought experiment:
  - Consider a tower on Earth
  - Shine a light ray from bottom to top
  - When light gets to top, turn its energy into mass
  - Then drop mass to bottom of tower, in Earth's gravity field
  - Then turn it back into energy
- Mass has clearly picked up speed and thus kinetic energy
- If we could do this, then we could get energy from nothing!

If mass and energy are interchangeable, then **light must feel gravity**.

Maxwell Equations





$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \epsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$$

- **No gravity** in Maxwell equations!
- Maxwell's equations are not exactly valid in reference frame of Earth's surface, where there is gravity
- The Earth's surface must **not be an inertial frame** of reference!

Participation: Recap #4

Respond to the poll on TurningPoint

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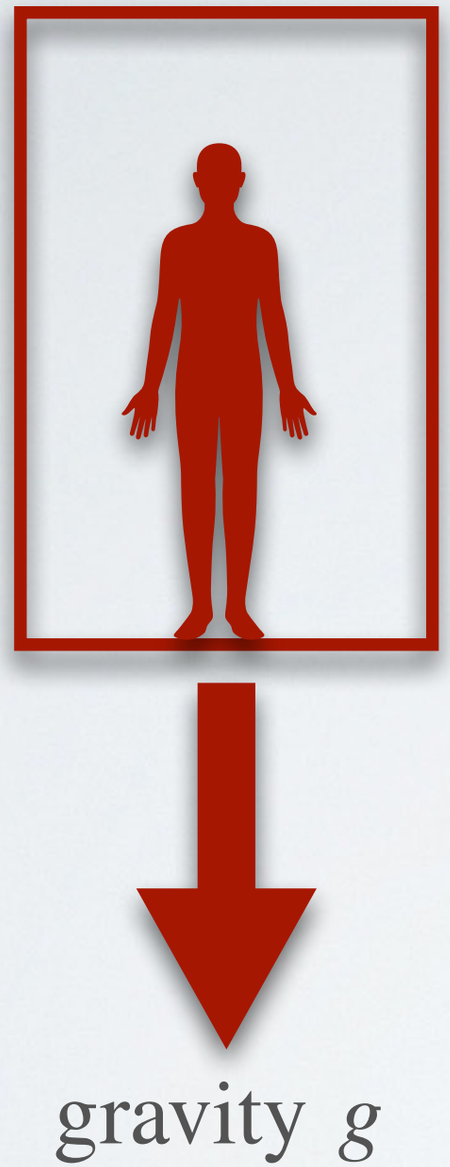
30 seconds

Weak equivalence principle

- Recall the “weak” equivalence principle
  - All objects accelerate at the same rate in a given gravitational field
  - Therefore, the inertial and gravitational masses must be the same
  - Verified experimentally, with fractional difference in masses $<10^{-13}$
- The effects of gravity and of inertial forces (fictitious forces associated with accelerated frames) cannot locally be distinguished

“Gravity is indistinguishable from any other acceleration”

Is gravity an inertial force?

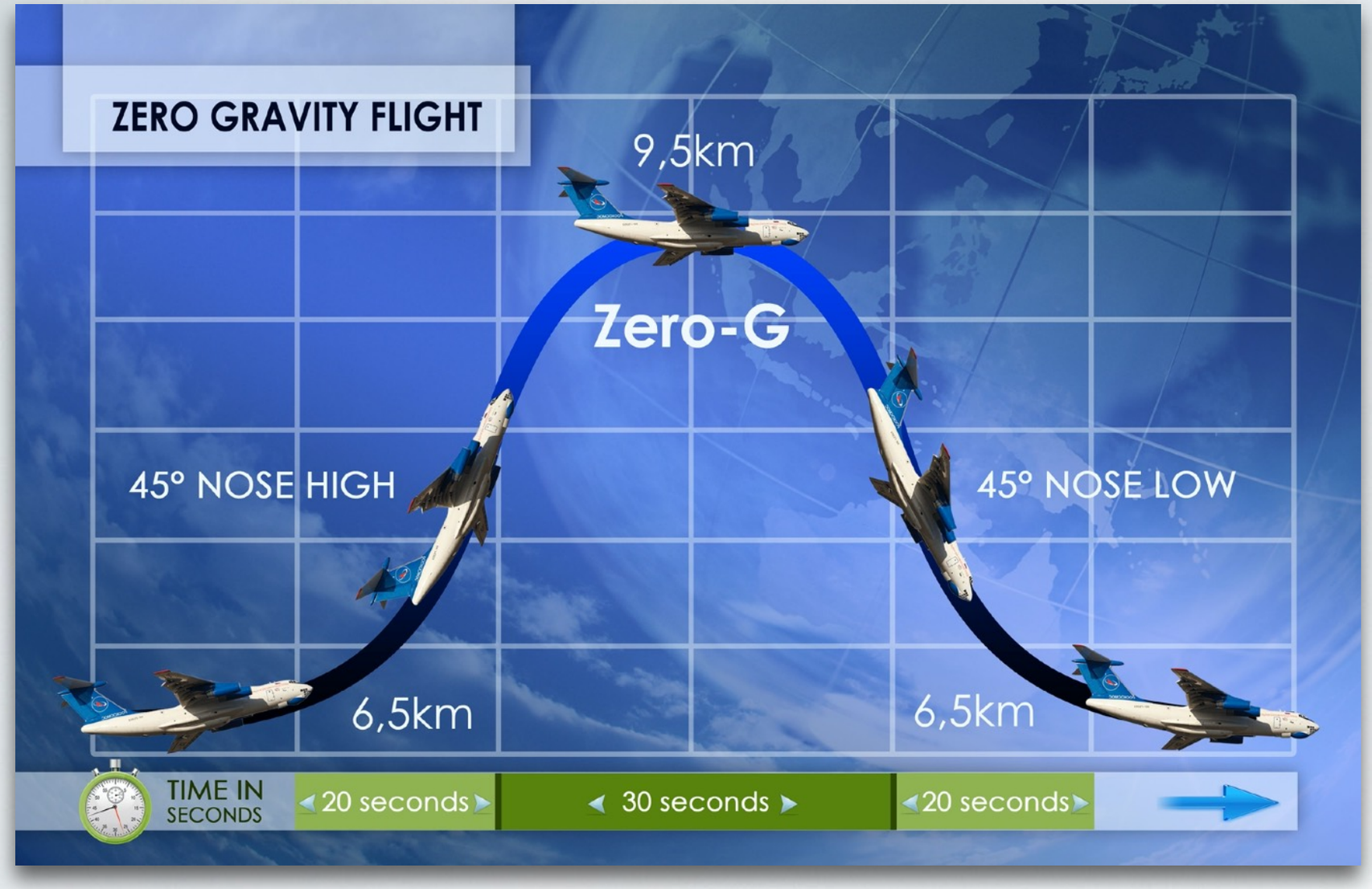


gravity g

acceleration a

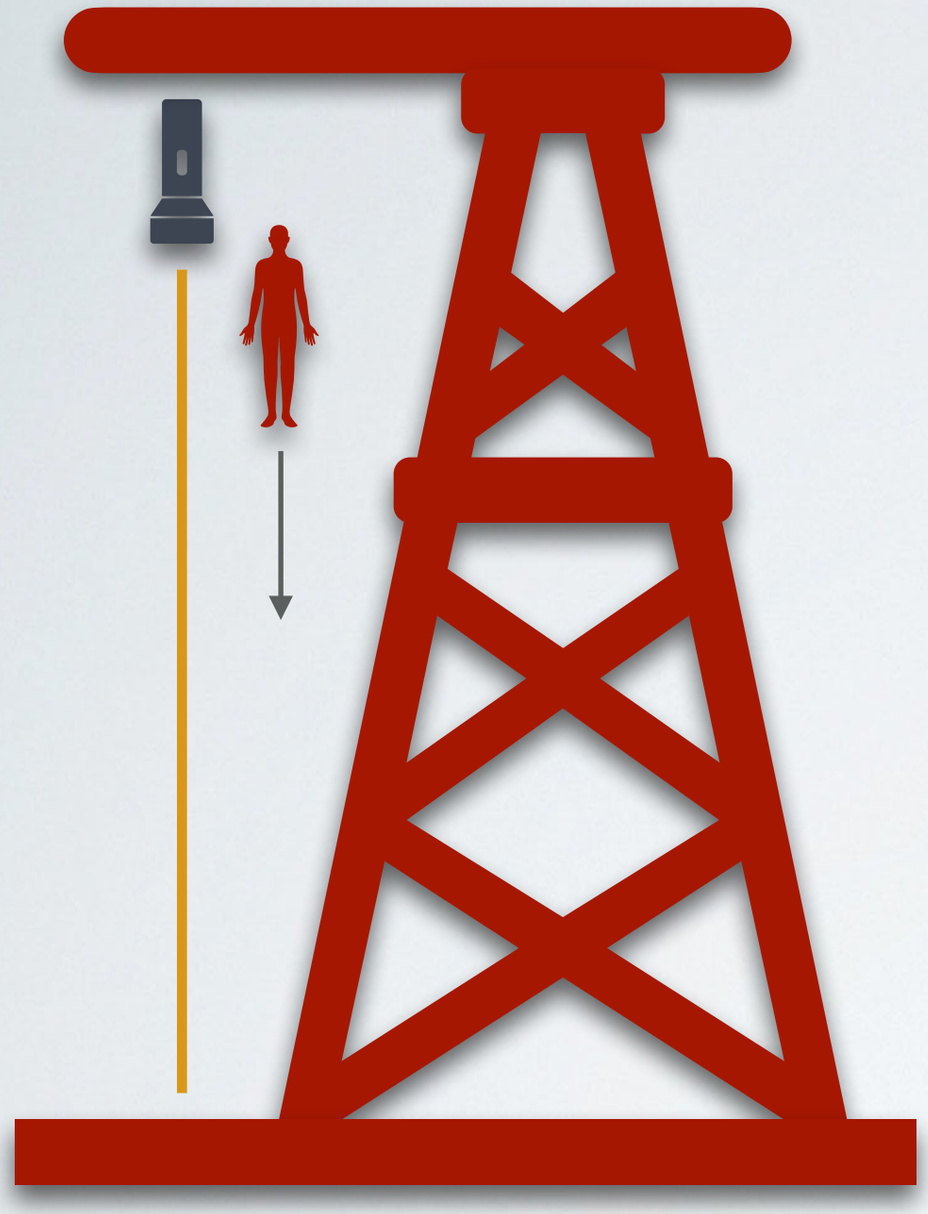
Elevator at rest on Earth equivalent to elevator being pulled by accelerating rocket in deep space

Free-falling frames



Free-falling: an object (or frame) in unrestrained motion under a (locally constant) gravitational field

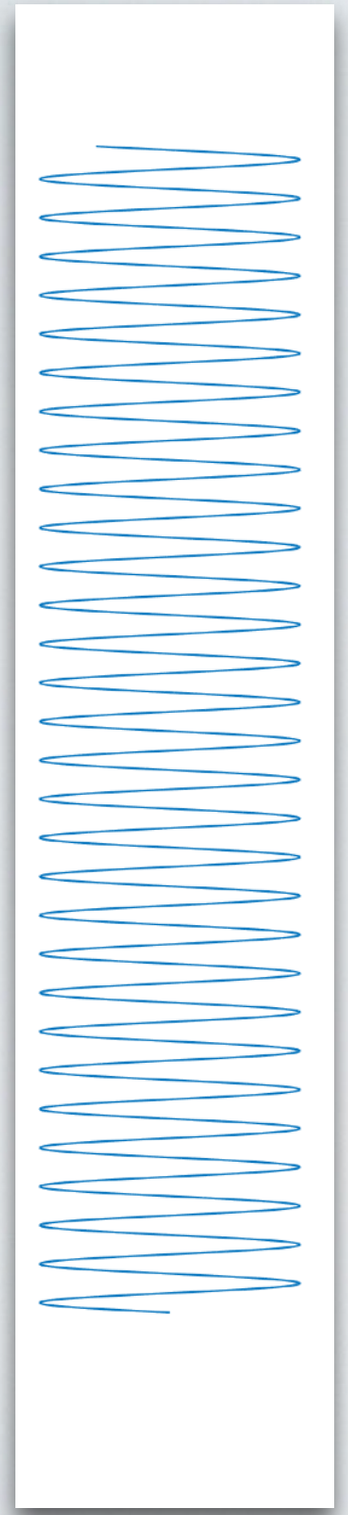
The tower problem #2



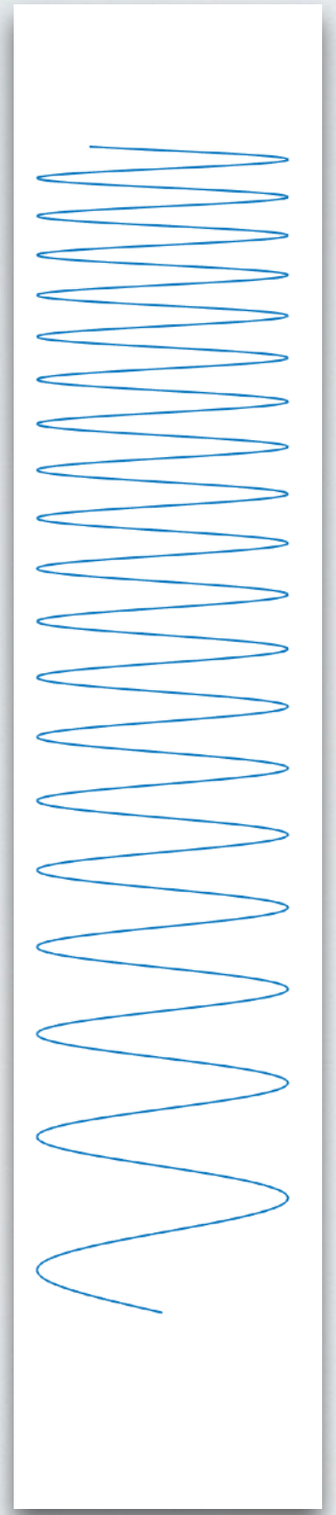
- Consider light ray aimed from top to bottom of tower
- **Free-falling observer** should see light ray travel **unaffected by gravity**, since free fall yields a state of apparent weightlessness
- From “Earth’s” frame…
  - Free-falling observer is traveling faster and faster
  - Falling observer would see an increasing **redshift** of light source according to special relativity

Without gravitational blueshift

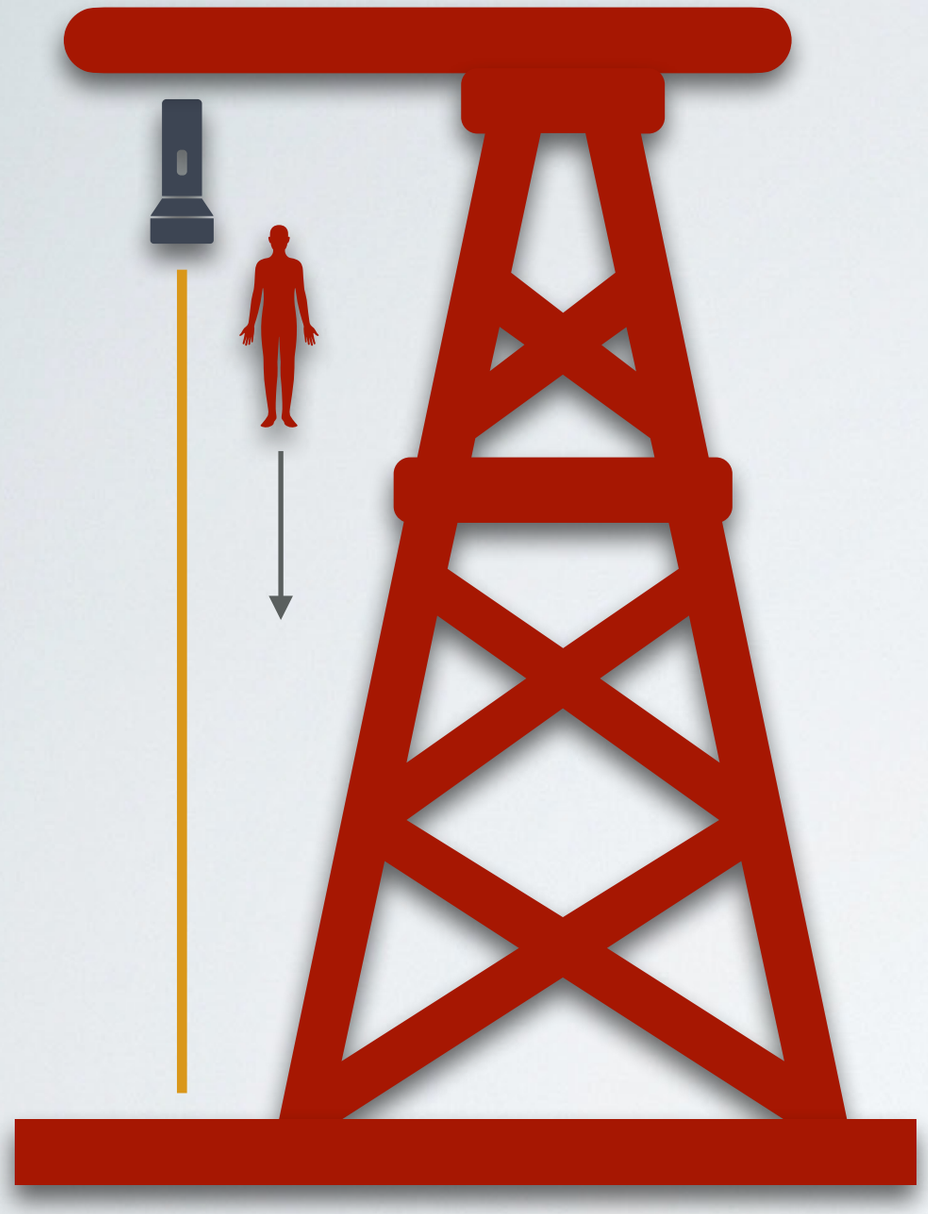
Photon frame:



Free-falling observer:



The tower problem #2



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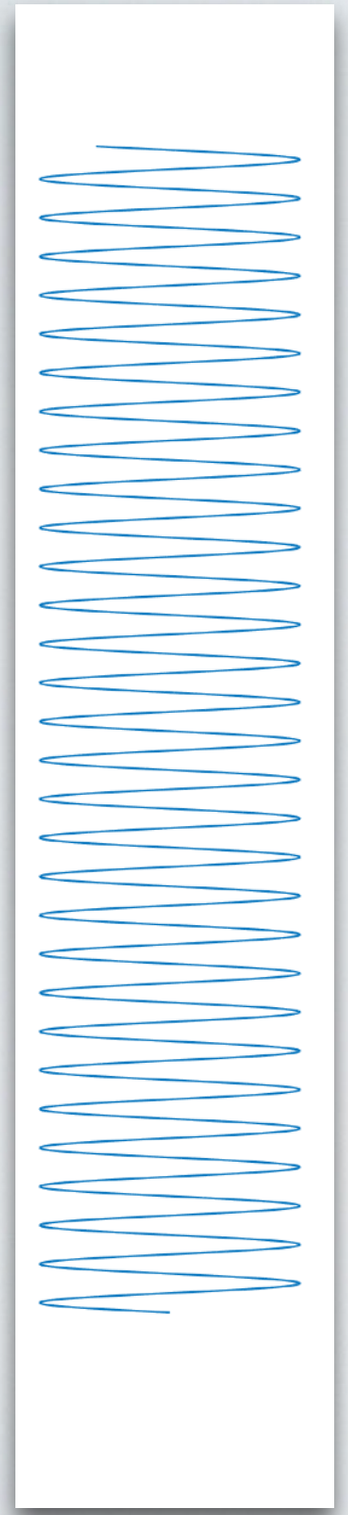
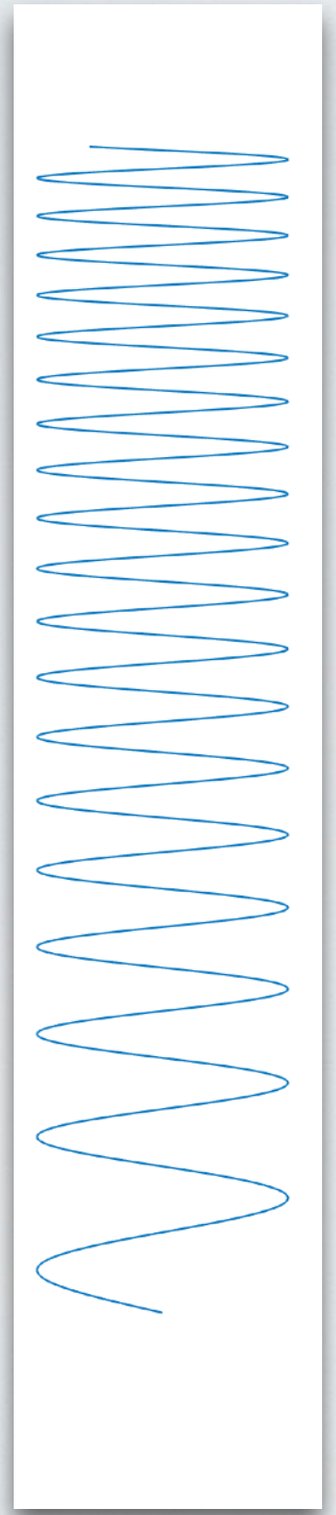
- From "Earth's" frame…
  - Free-falling observer is traveling faster and faster
  - Falling observer would see an increasing **redshift** of light source according to special relativity
- If free-falling observer is to see a constant frequency light beam, then **light must get blueshifted** (in Earth frame) as it falls in gravitational field

Without gravitational blueshift

Photon frame:

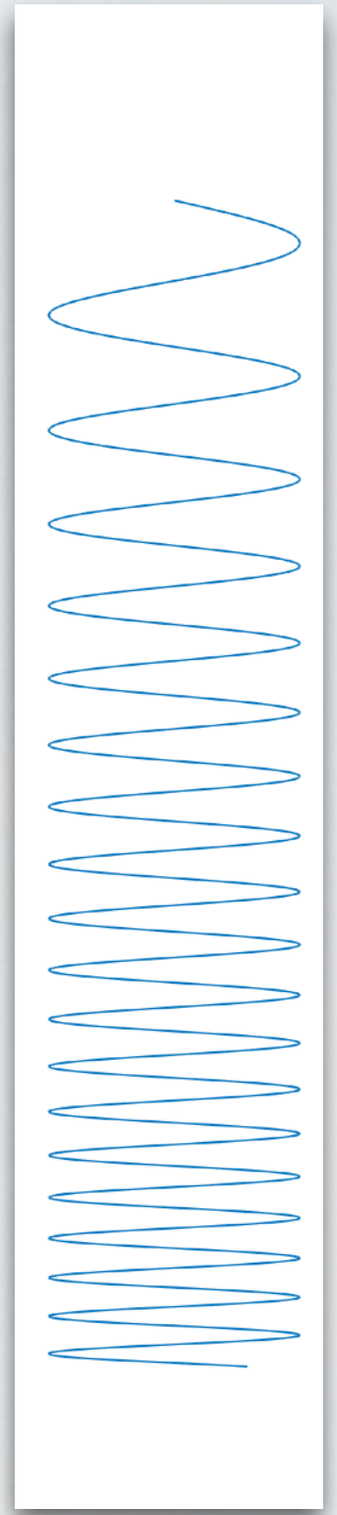


Free-falling observer:

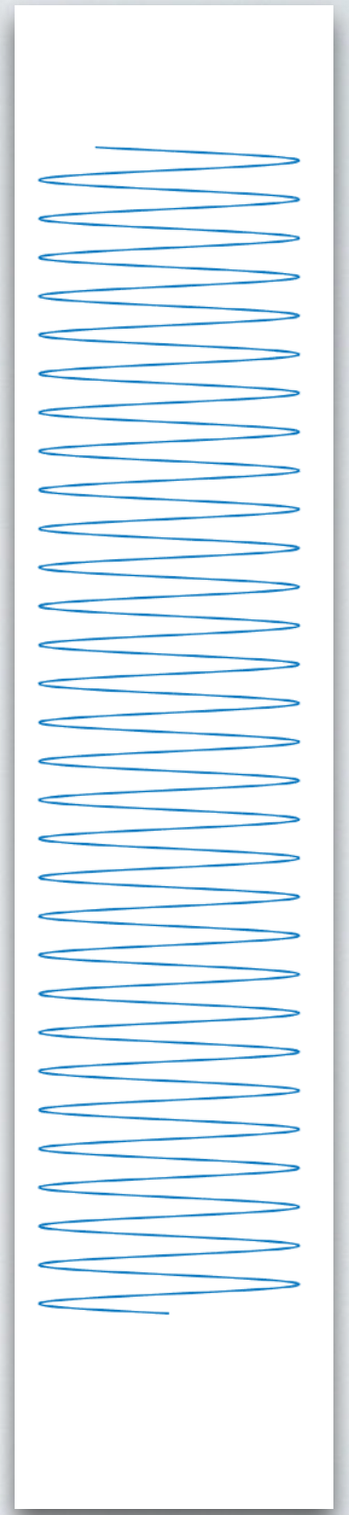


With gravitational blueshift

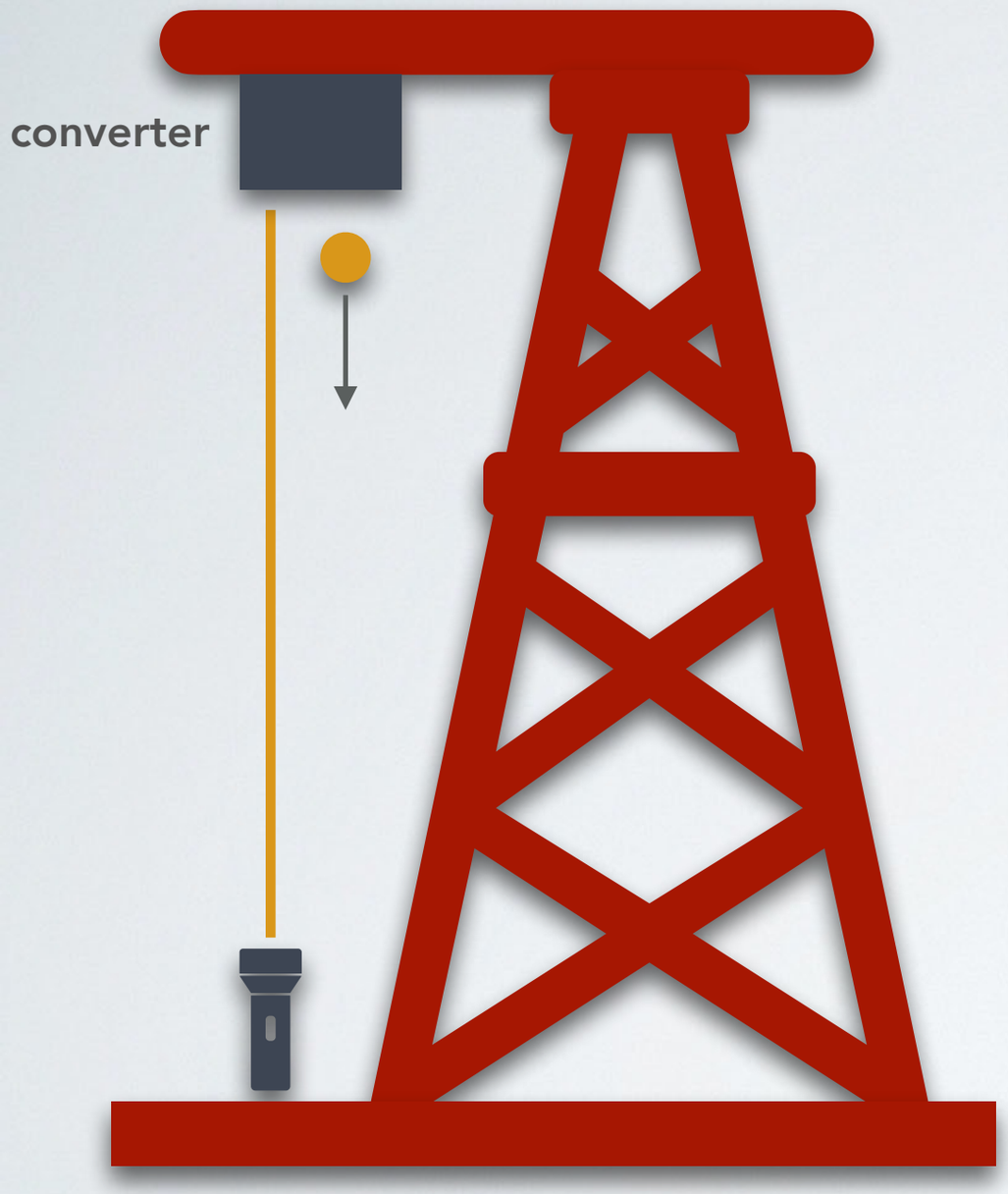
Photon frame:



Free-falling observer:



The tower problem #1



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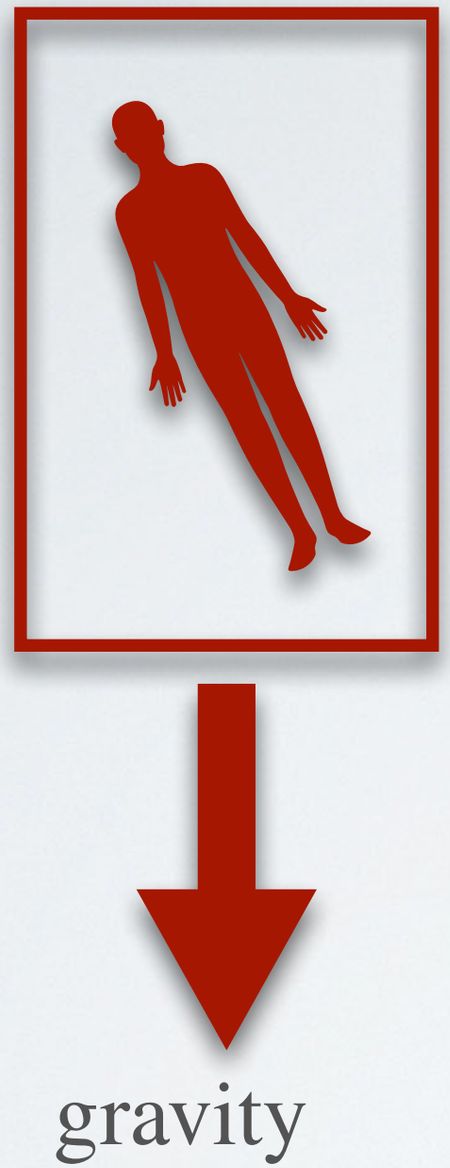
- From "Earth's" frame…
  - Free-falling observer is traveling faster and faster
  - Falling observer would see an increasing **redshift** of light source according to special relativity
  - If free-falling observer is to see a constant frequency light beam, then **light must get blueshifted** (in Earth frame) as it falls in gravitational field
- Light beam aimed upward must conversely be increasingly redshifted with height
- **Gravitational redshifting** removes just the right amount of energy to solve the tower paradox!

Strong (Einstein) equivalence principle

The laws of physics are the same in all inertial and free-falling frames of reference. Such frames cannot be distinguished by local experiments.

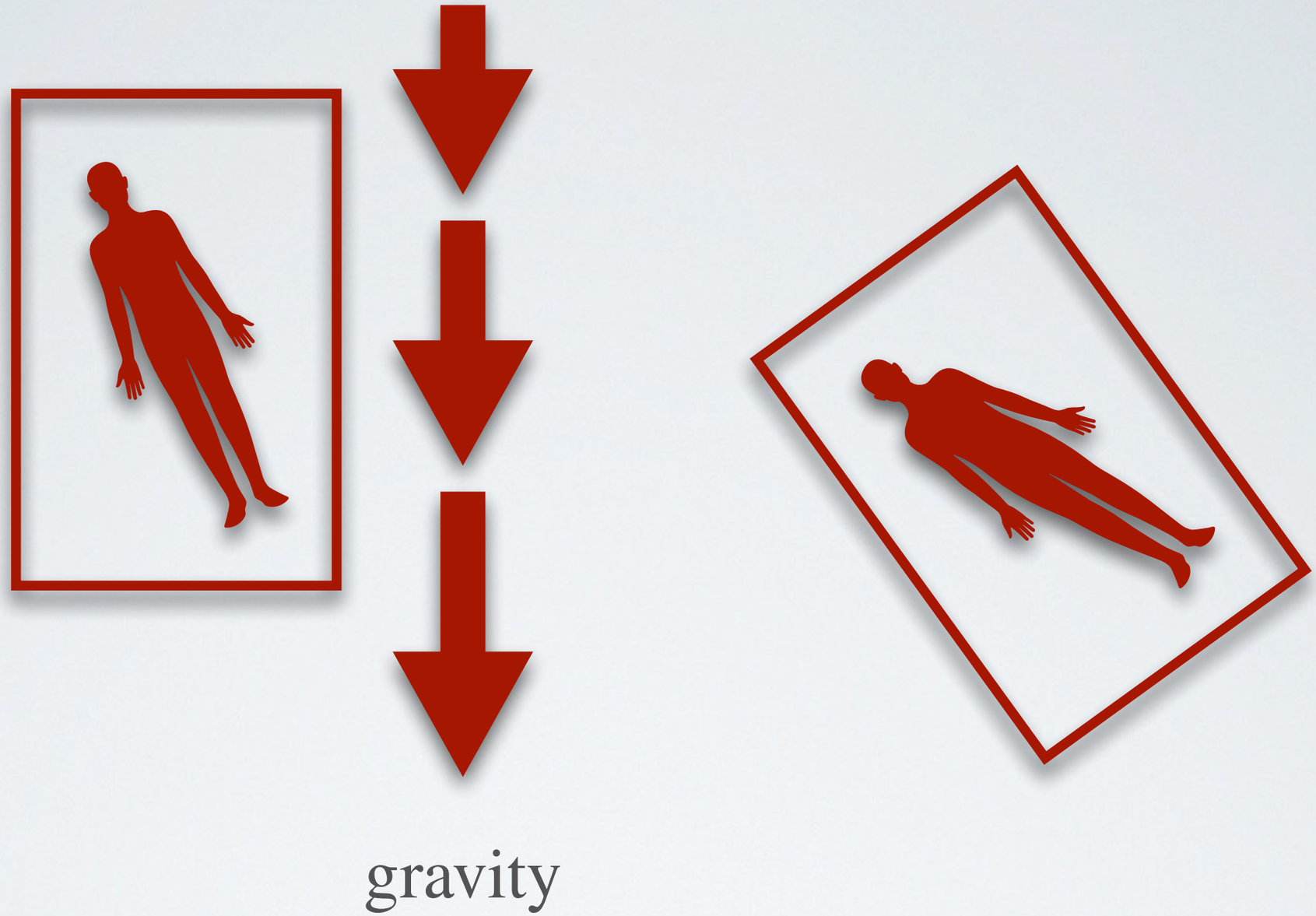
- Extends relativity to **free-falling reference frames**
- Free-falling: an object (or frame) in unrestrained motion under a **locally constant gravitational field**
- Frames are now **local**, i.e., they refer to a particular location

Strong (Einstein) equivalence principle



Interior of elevator free-falling on Earth is equivalent to interior of elevator floating freely in deep space

Strong (Einstein) equivalence principle



gravity

Gravitational field must be locally constant!

Participation: Class activities survey

Take the (anonymous) survey on Canvas!

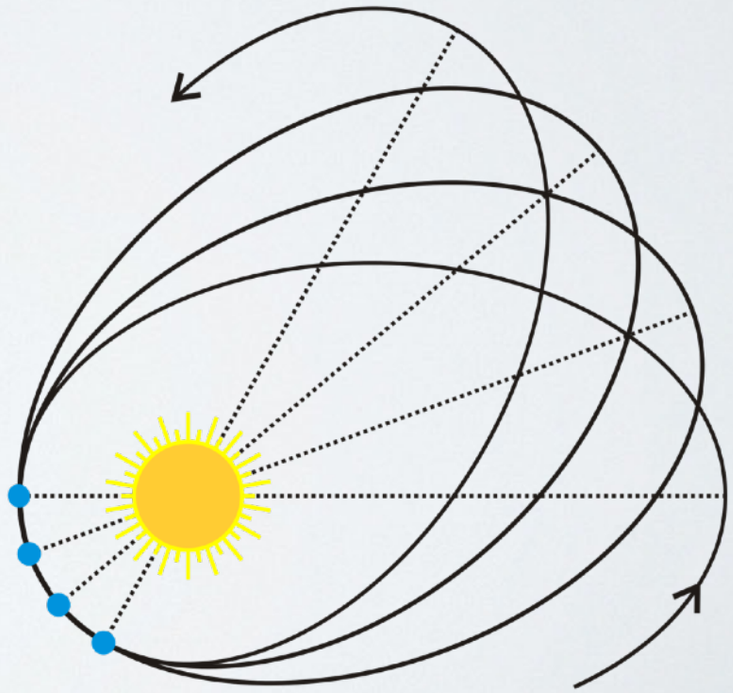
 5 minutes

Part 3: The basics of General Relativity

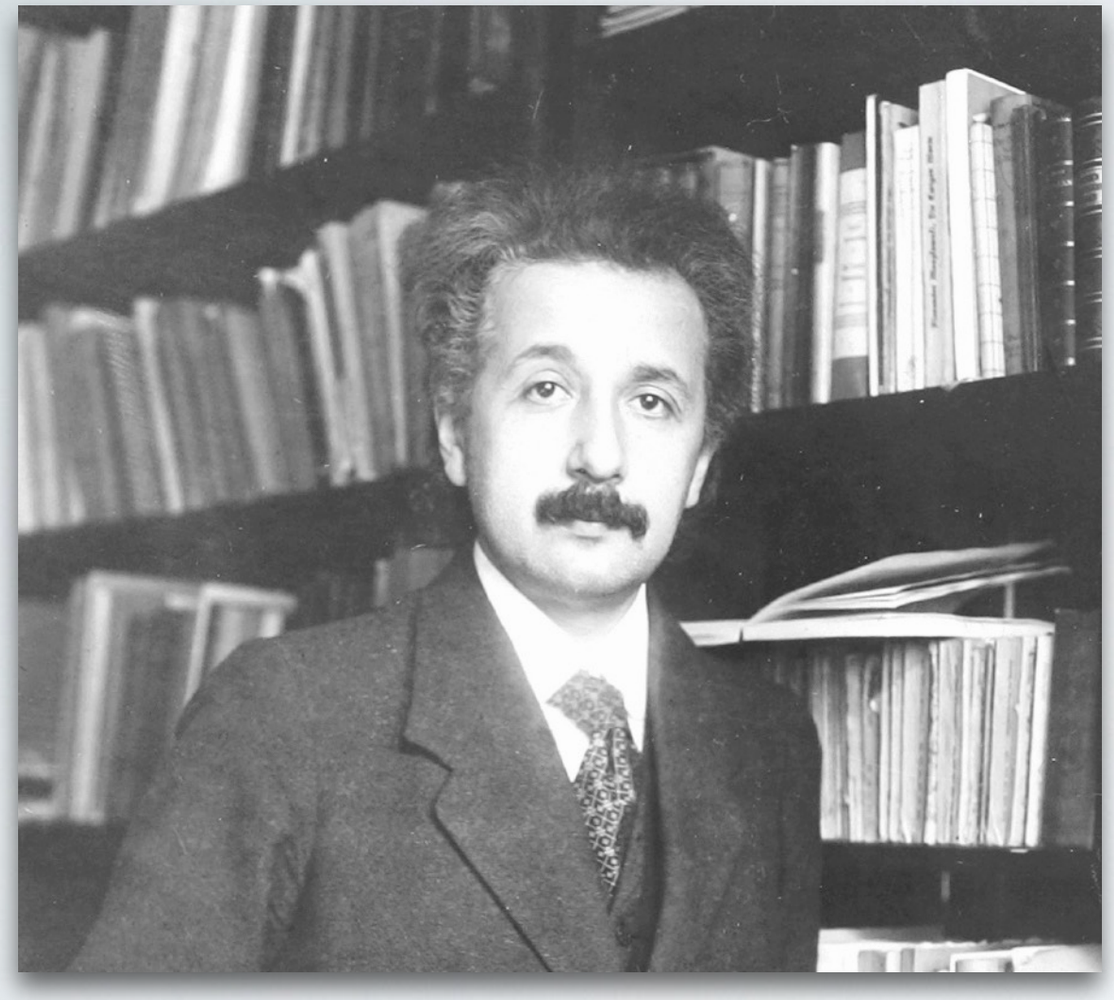
Why General Relativity?

- Tower problem demonstrates need for strong equivalence principle
- Issues with Special Relativity
  - Relies on inertial frames, but how are these frames related to matter?
- Issues with Newtonian Gravity
  - Instantaneous force communication is incompatible with SR
  - In whose frame do we measure r? Does the force depend on the frame?
  - Perihelion drift of Mercury

$$F = \frac{GMm}{r^2}$$

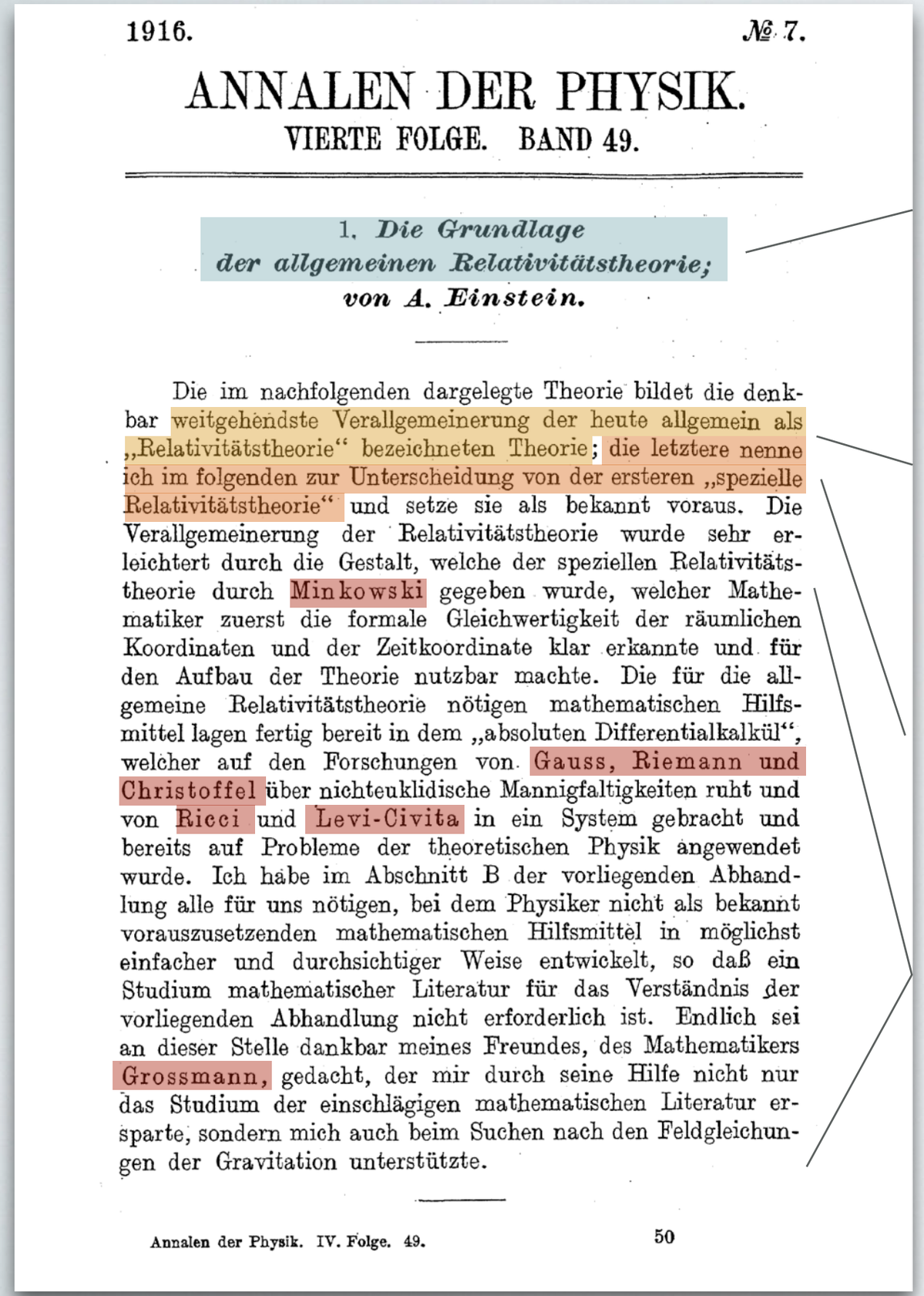


Albert Einstein (1879 - 1955)



- Groundbreaking work in many areas of physics
- PhD in 1905 (unrelated to relativity)
- Professor in Bern, Prague, Berlin
- Nobel prize in 1921
- Fled Germany to the US in 1933
- Spent rest of his career at Princeton

General relativity (1915/16)



1916. № 7.

ANNALEN DER PHYSIK.

VIERTE FOLGE. BAND 49.

1. *Die Grundlage der allgemeinen Relativitätstheorie;*

von A. Einstein.

Die im nachfolgenden dargelegte Theorie bildet die denkbar weitgehendste Verallgemeinerung der heute allgemein als „Relativitätstheorie" bezeichneten Theorie; die letztere nenne ich im folgenden zur Unterscheidung von der ersteren „spezielle Relativitätstheorie" und setze sie als bekannt voraus. Die Verallgemeinerung der Relativitätstheorie wurde sehr erleichtert durch die Gestalt, welche der speziellen Relativitätstheorie durch Minkowski gegeben wurde, welcher Mathematiker zuerst die formale Gleichwertigkeit der räumlichen Koordinaten und der Zeitkoordinate klar erkannte und für den Aufbau der Theorie nutzbar machte. Die für die allgemeine Relativitätstheorie nötigen mathematischen Hilfsmittel lagen fertig bereit in dem „absoluten Differentialkalkül", welcher auf den Forschungen von Gauss, Riemann und Christoffel über nichteuklidische Mannigfaltigkeiten ruht und von Ricci und Levi-Civita in ein System gebracht und bereits auf Probleme der theoretischen Physik angewendet wurde. Ich habe im Abschnitt B der vorliegenden Abhandlung alle für uns nötigen, bei dem Physiker nicht als bekannt vorauszusetzenden mathematischen Hilfsmittel in möglichst einfacher und durchsichtiger Weise entwickelt, so daß ein Studium mathematischer Literatur für das Verständnis der vorliegenden Abhandlung nicht erforderlich ist. Endlich sei an dieser Stelle dankbar meines Freundes, des Mathematikers Grossmann, gedacht, der mir durch seine Hilfe nicht nur das Studium der einschlägigen mathematischen Literatur ersparte, sondern mich auch beim Suchen nach den Feldgleichungen der Gravitation unterstützte.

Annalen der Physik. IV. Folge. 49. 50

The foundation of the general theory of relativity

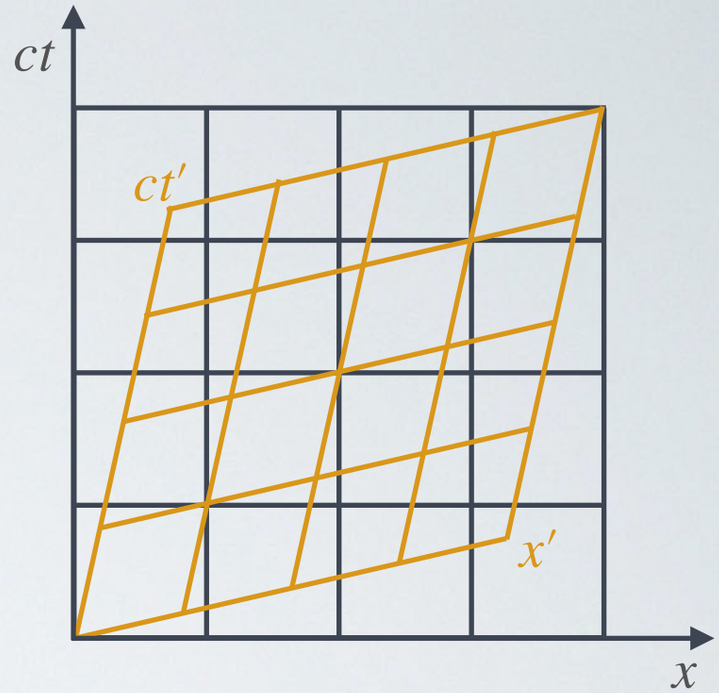
… greatest possible generalization of the theory known as "relativity theory"

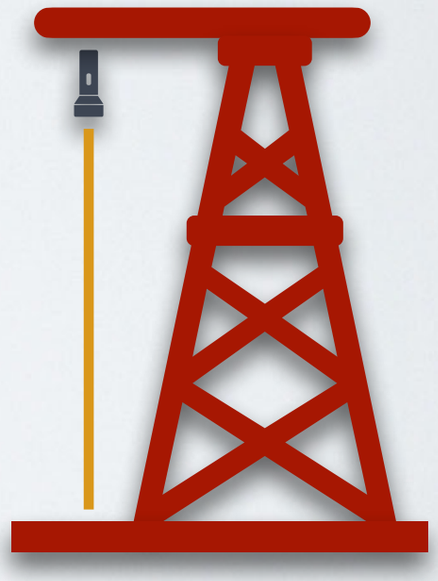
… to distinguish them, will call the latter "special theory of relativity"

Acknowledges many other scientists

General relativity

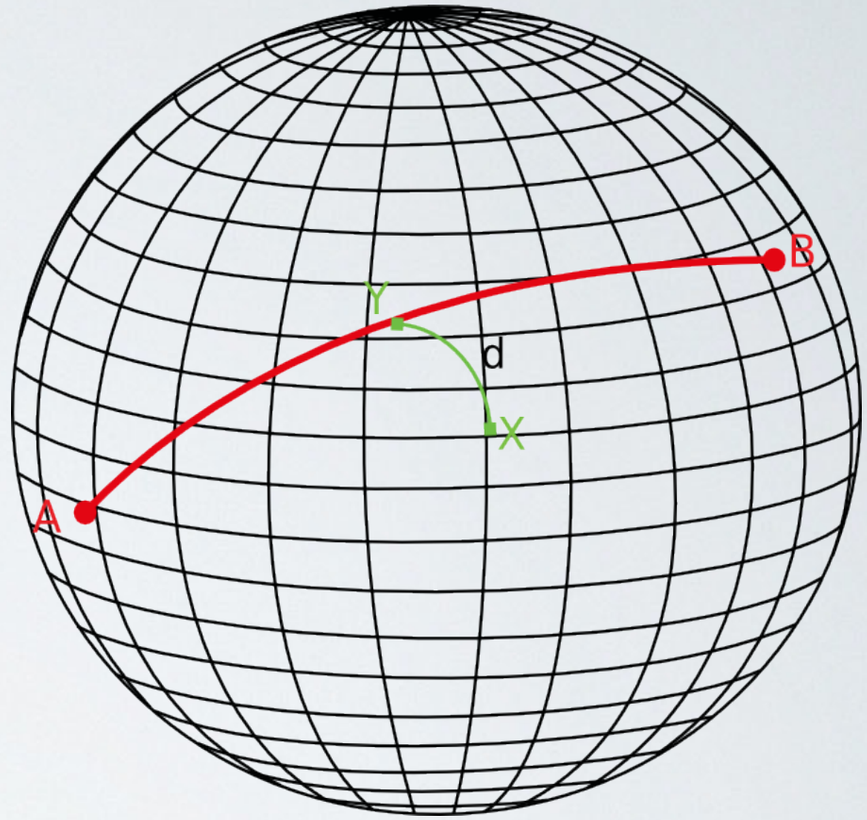
- Success of Special Relativity showed that space and time are closely interlinked
- The “tower thought experiment” suggested that free-fall observers are (locally) free of effects of gravity
- What if gravity is an illusion caused by the fact that we live in an accelerating frame?
- Accelerating frames are like inertial frames only locally; how are they combined for different accelerations in different directions?



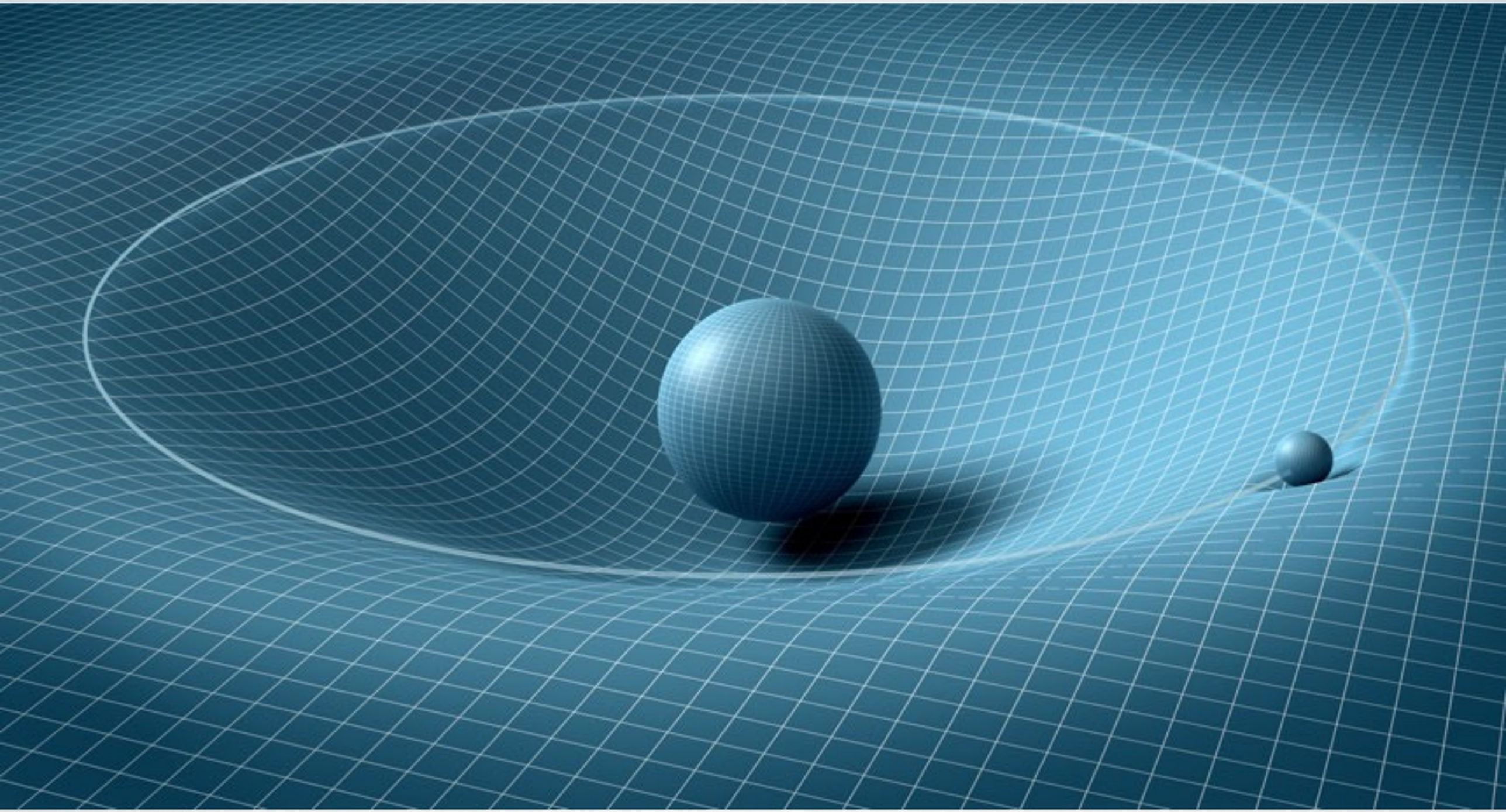


General relativity

- Einstein's proposal
  - 4-dimensional space-time is **curved**, not flat
  - Free-falling objects **move on shortest paths** through curved space-time ("geodesics")
- What is a geodesic?
  - Generalization of a straight line to curved space
  - **Shortest path** between two points on a surface (e.g., path flown by an aircraft between cities on the globe)
- Mass/energy does not cause gravitational force but instead **curves space-time**

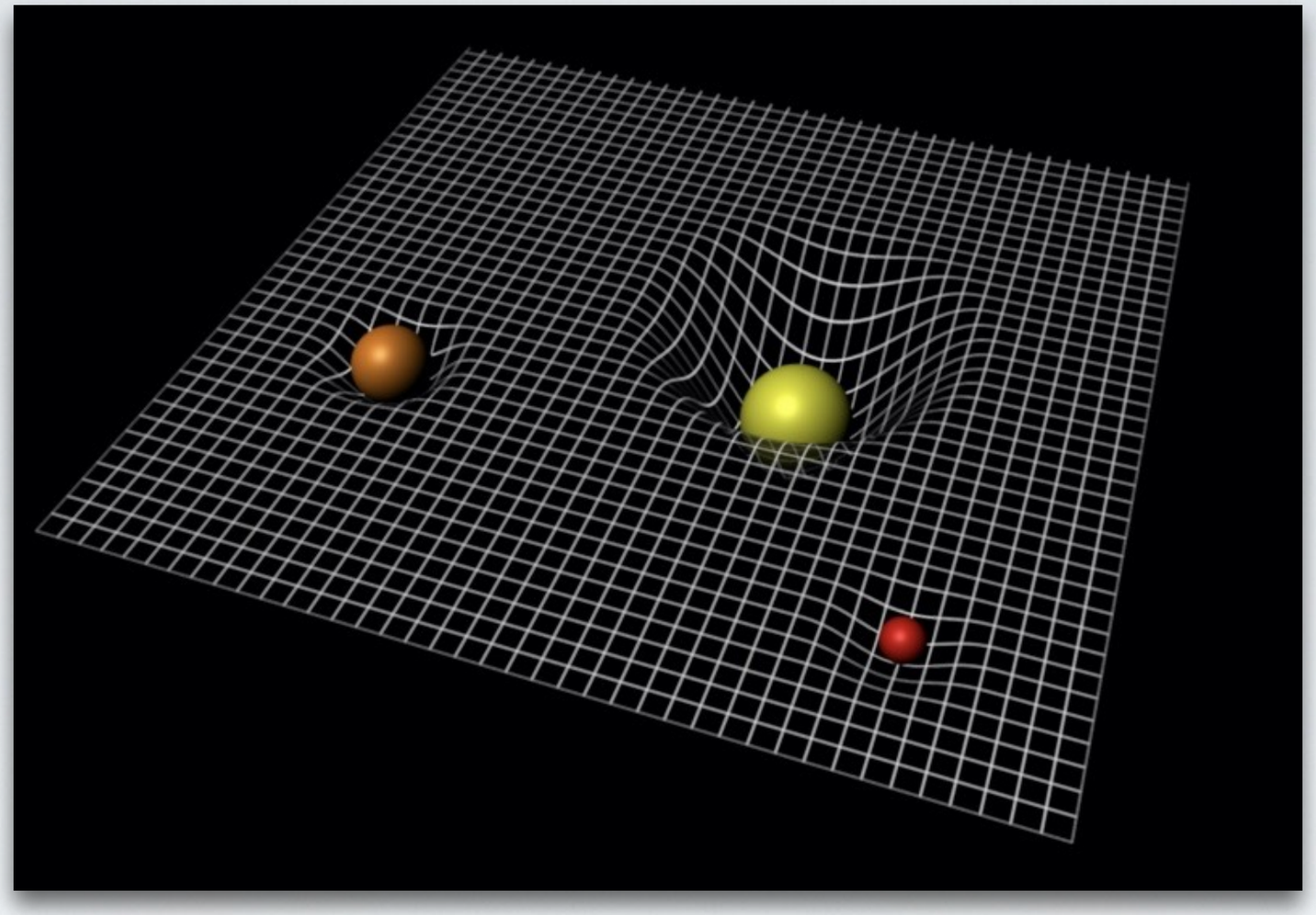


General relativity



Artist’s impression!

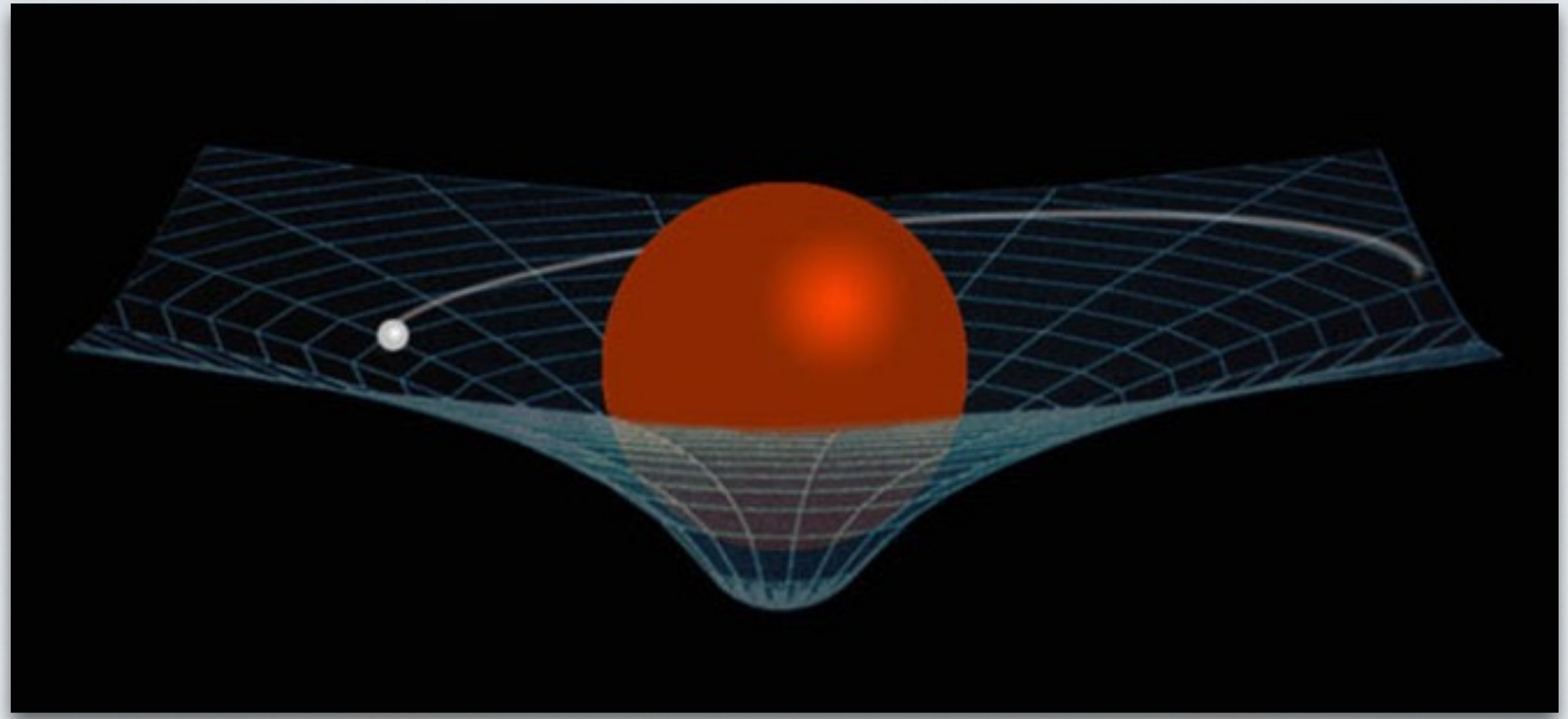
Warped spacetime

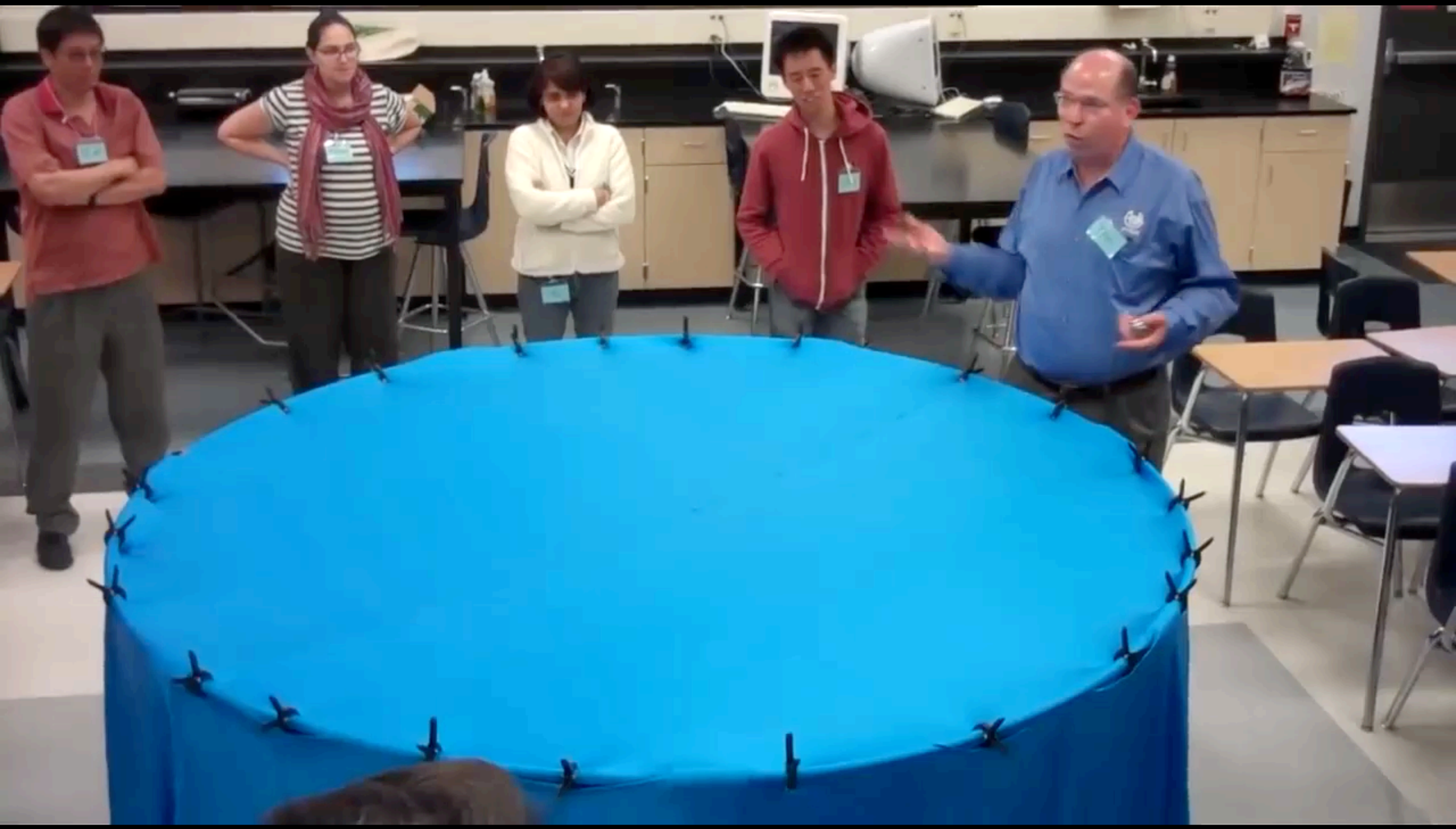


- Two-dimensional space as an analogy: rubber sheet with weights
- Amount that sheet sags depends on how heavy weight is
- Lines that would be **straight** become **curved (to external observer)** when sheet is “weighted”

Orbits in warped spacetime

- Marble would follow straight line if weight were not there
- Marble’s orbit becomes curved path because weight warps space





youtube.com/watch?v=MTY1Kje0yLg

General relativity

- Within free-falling frames of reference, Special Relativity applies

- Free-falling particles or observers move on geodesics (shortest paths) through curved space-time

- The distribution of matter and energy determines how space-time is curved

Space-time curvature tells matter/energy how to move, matter/energy tells space-time how to curve

Einstein field equation

Newton's gravitational constant

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

"Curvature tensor" that describes the curvature of 4D space-time

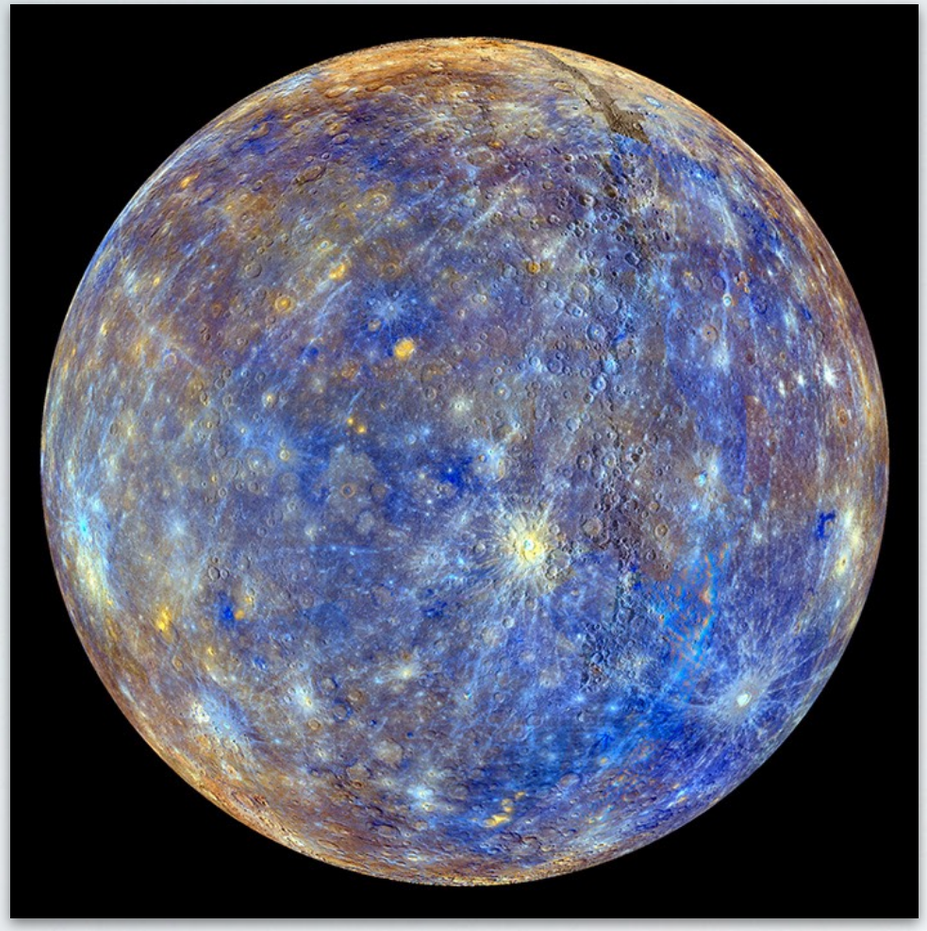
speed of light

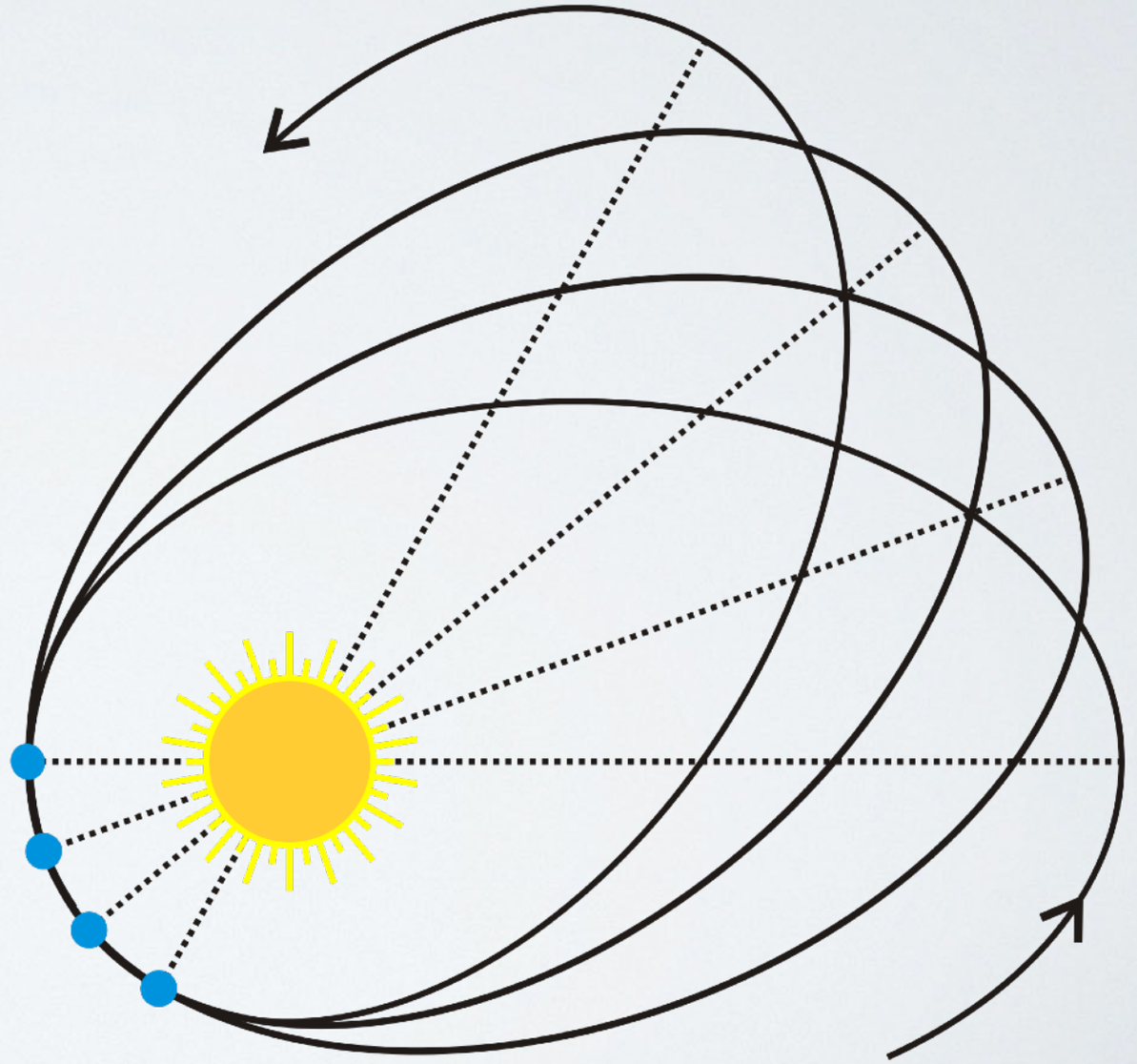
"Stress-energy tensor" that describes distribution of mass and energy

- Geometry = constant * (matter + energy)
- G and T can be written in terms of **components**, similar to a matrix
- Horrendous set of **10 coupled equations!**
- For weak gravitational fields, this **reduces to Newton's law** of gravitation, to an excellent approximation

Part 4: Observational tests: Perihelion precession

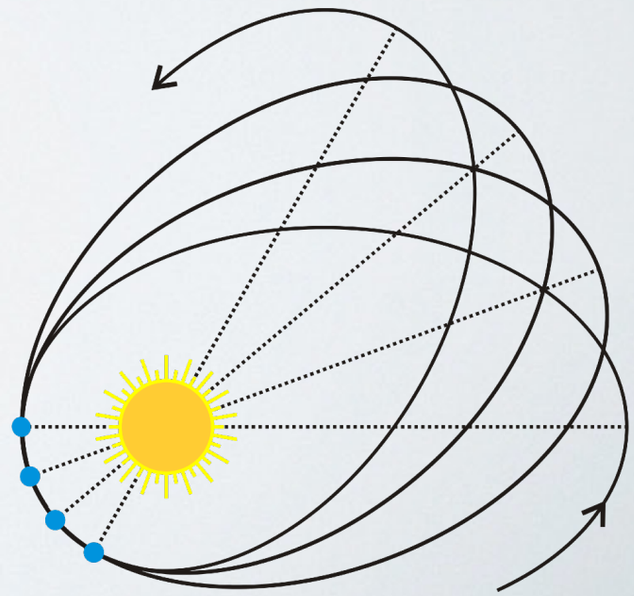
Perihelion precession of Mercury





Perihelion precession of Mercury

- This happens to all planets, but **Mercury is closest to the Sun** (a place with unusually bendy space-time in the Solar system)
- Effect is small: orbit twists by 5600 arc-seconds (1.56 degrees) per century
- Newtonian gravity can explain 5557 arcsec / century due to
  - Changes in Earth's frame (equinox precession): 5025.6 arcsec
  - Gravitational effect of other planets: 531.4 arcsec
  - Deformation of the Sun: 0.025 arcsec
- But this still leaves **43 arcsec / century** unexplained
- Using GR, Einstein **predicted** (with no fiddling) that Mercury's orbit should precess exactly 43 extra arcsec / century!



Take-aways

- Light is affected by gravity, which implies that the surface of **Earth cannot be an inertial frame**

- The **strong equivalence principle** extends relativity to free-falling frames of reference

- Free-falling objects or observers move on shortest paths (**geodesics**) through curved space-time

- The **curvature of space-time** is determined by matter and/or energy

Next time…

We'll talk about:

- Curved spacetime, light bending, time dilation

Assignments

- Post-lecture quiz (by tomorrow night)
- Homework #2 (due 10/07)

Reading:

- H&H Chapter 8