# ASTR 340: Origin of the Universe

Prof. Benedikt Diemer

## Lecture 12 • The Expanding Universe

10/07/2021

# Messier catalog discussion

Compact, round cluster

- Circle shaped + yellow/white/blue
- Pediodt!
- Bright center dot
- Circle worlds
- Fuzzballs




Scattered stars

- Paint splatter
- Sprinkles
- Cluster worlds
- Less / mildly / super dense
- Blue dots

Gas clouds

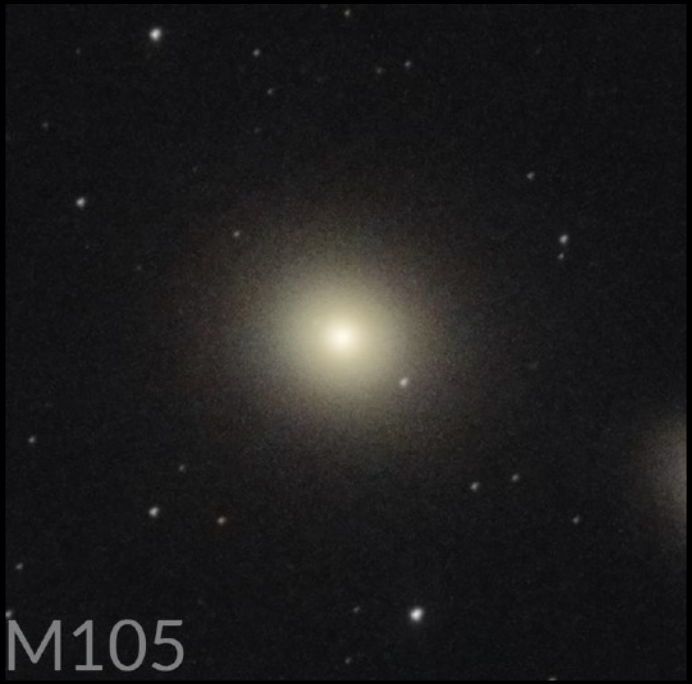
- Starry mist
- Cloudy Wowdies
- Space clouds
- Dispersed cloud objects
- Explosions




Strong colors

- RED >:)
- Vibrant colors
- Red / yellow / blue



Spirals

- Swirly Wirlies
- Spiral glowing lights




Ellipsoids

- Frisbee Wisbees
- Oval worlds
- Disks





Lefty's Astrophotography

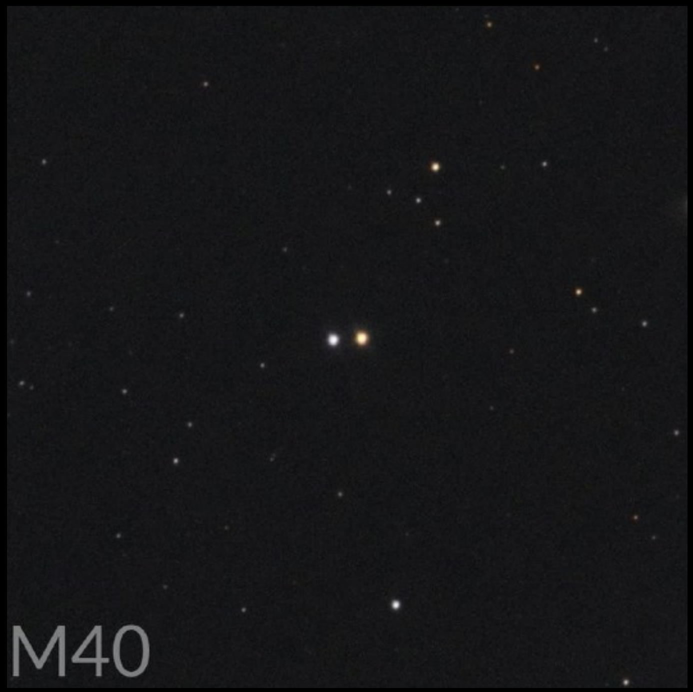
Elongated

- Lines
- Line-like disks


Lefty's Astrophotography

Weirdos

- Looks exploded



**Modern, scientific categories**

- Globular cluster (round, compact star cluster)
- Open cluster (less compact cluster of stars)
- Gas cloud (diffuse nebula)
- Planetary nebula (gas cloud around single star)
- Supernova remnant (leftovers of stellar explosion)
- Double star (two stars that happen to appear close)
- Galaxy (the only objects not in MW)

In the Milky Way

# Globular clusters (29)




# Open clusters (26)




Lefty's Astrophotography

# Gas clouds (7)

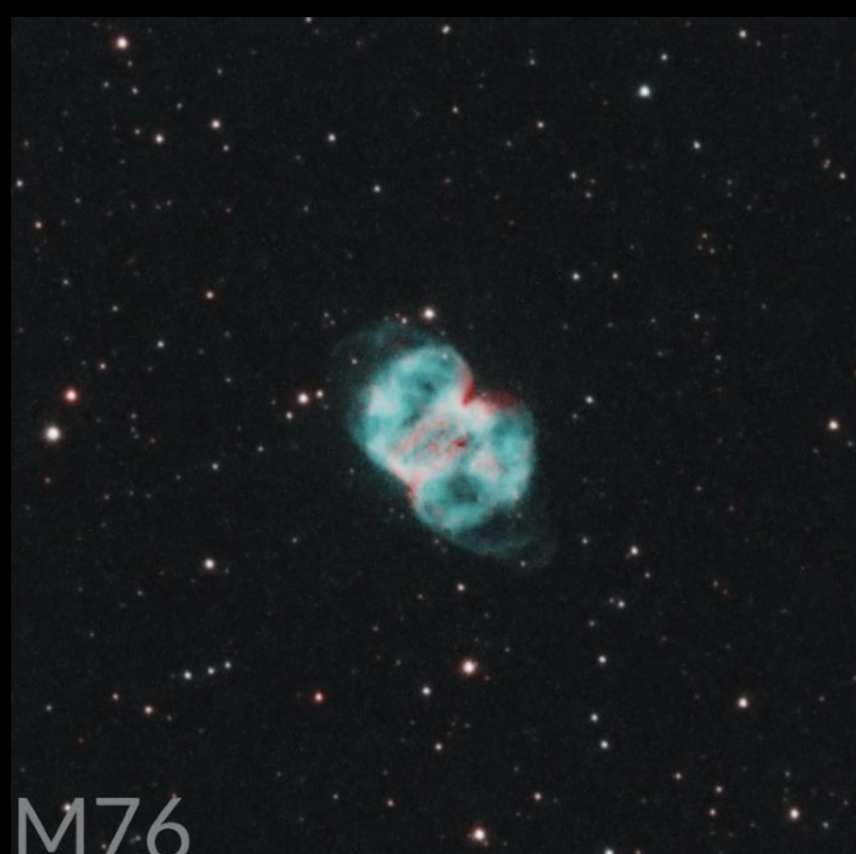




Lefty's Astrophotography

# Planetary nebulae (4)





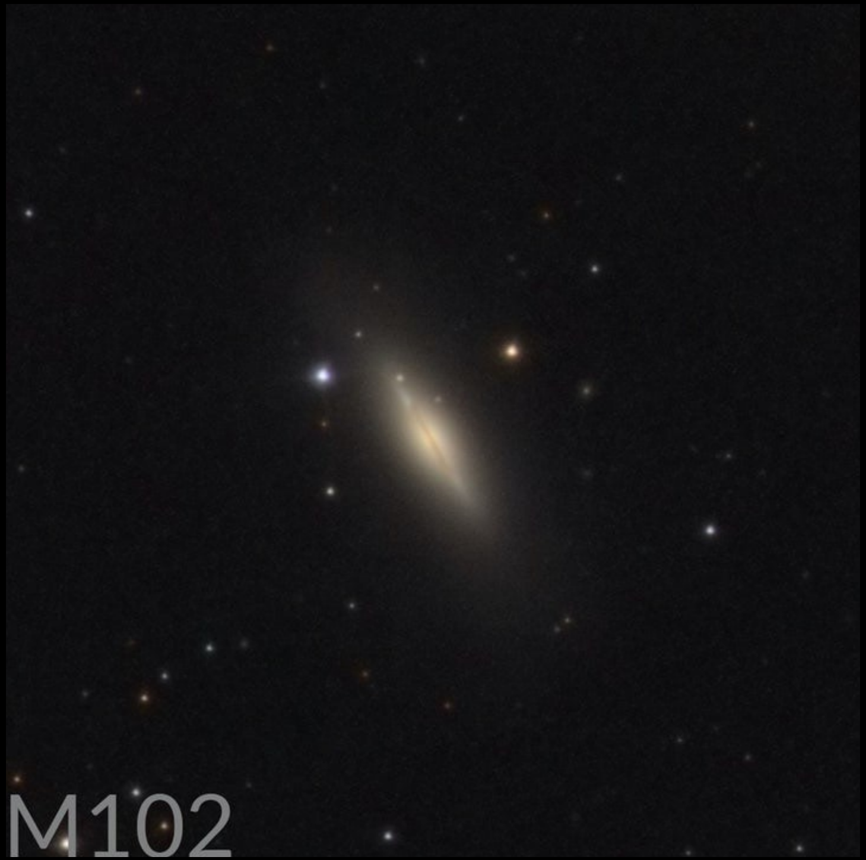


Lefty’s Astrophotography

# Supernova remnant (1)

# Double star (1)

# Galaxies (40)






Lefty's Astrophotography

# Part 0: Recap

# Participation: Recap #1

**TurningPoint:**
What is special about Cepheid variable stars?

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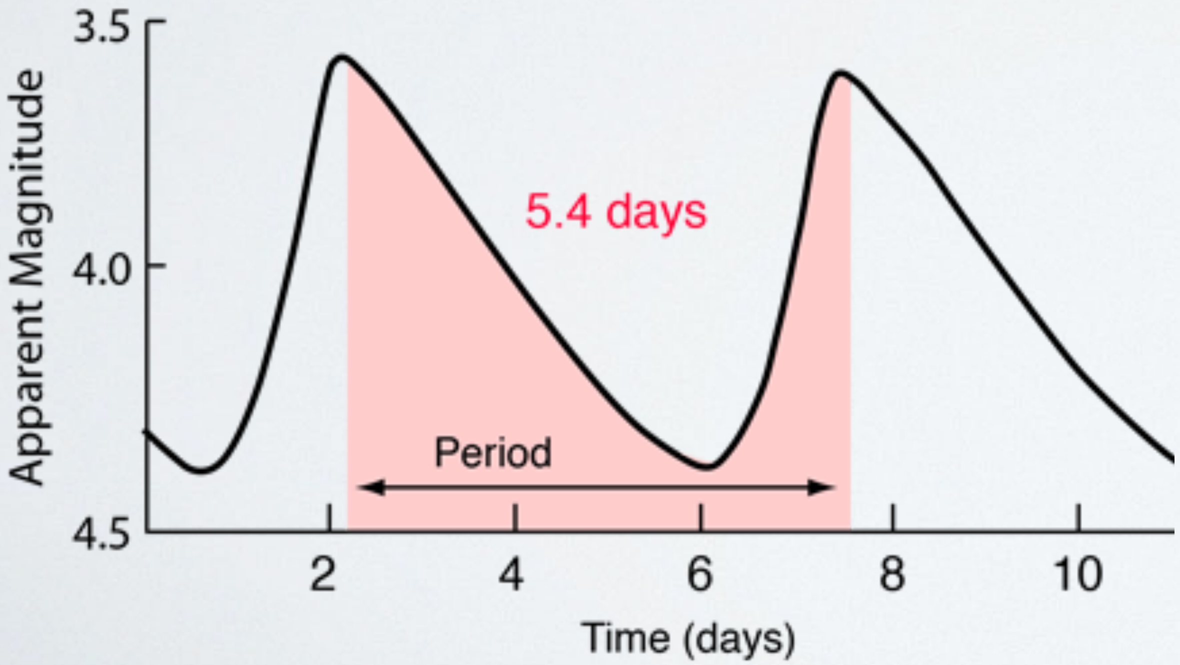
30 seconds

# The first standard candle: Cepheid Variables

- In 1912, Henrietta Swan Leavitt observed a type of **variable star** called Cepheids
- Instrinsic luminosity can then be obtained from apparent brightness and **parallax distance**
- She discovered that Cepheids' total luminosity is related to the **period of fluctuations**
- Cepheids can be used as **standard candles!**



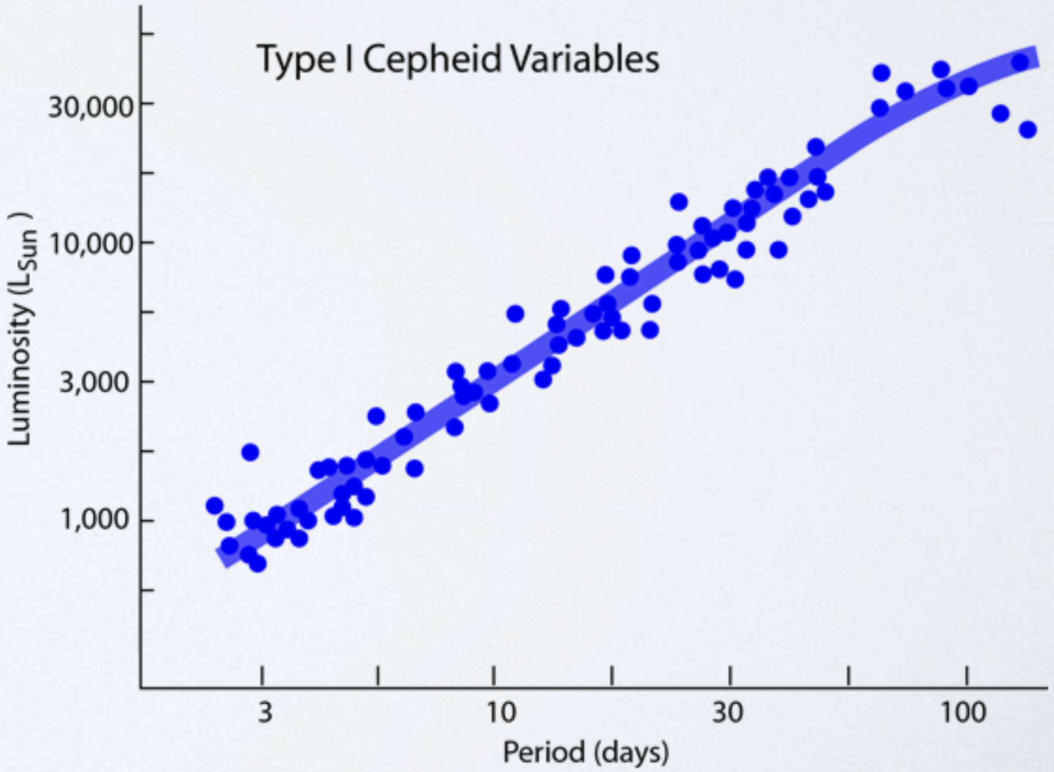




Image: hyperphysics.phy-astr.gsu.edu

# Participation: Recap #2

**TurningPoint:**
What causes cosmological redshift?

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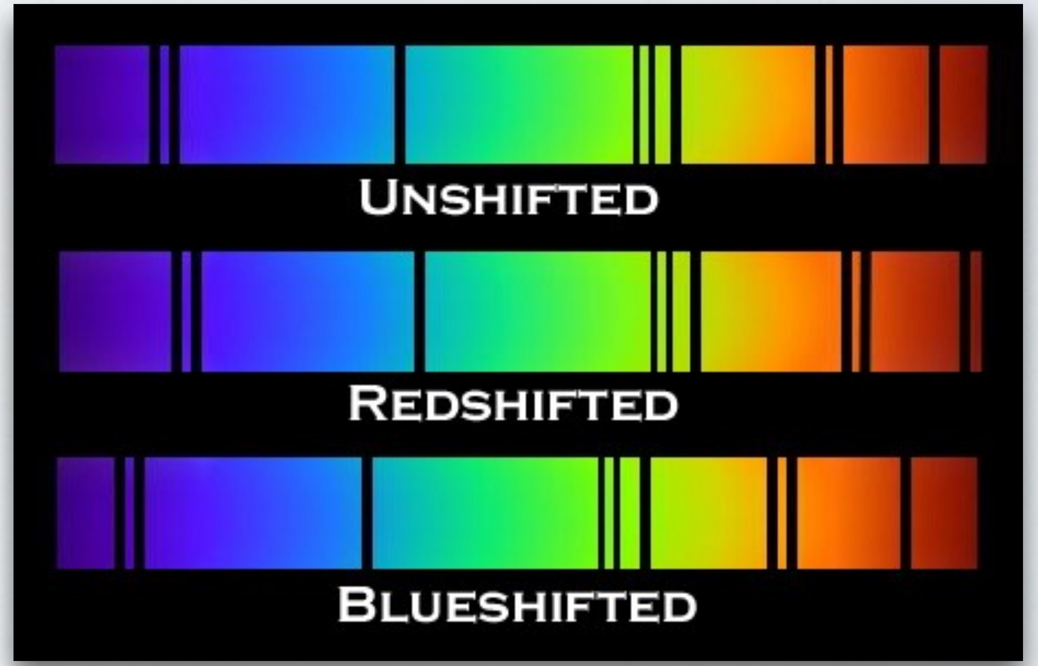
30 seconds

# Cosmological Redshift

$$z = \frac{\lambda_{\text{obs}} - \lambda_{\text{em}}}{\lambda_{\text{em}}}$$

- Defined as the **relative shift in wavelength** between the emitted and observed light
- The "redshift" can be…
  - **z > 0:** redder, longer wavelength
  - **z < 0:** bluer, shorter wavelength
- Approximation $z \approx v/c$ can be used at **low redshift** and small velocities (v << c)



# Participation: Recap #3

**TurningPoint:**
What does the Hubble-Lemaitre law say?

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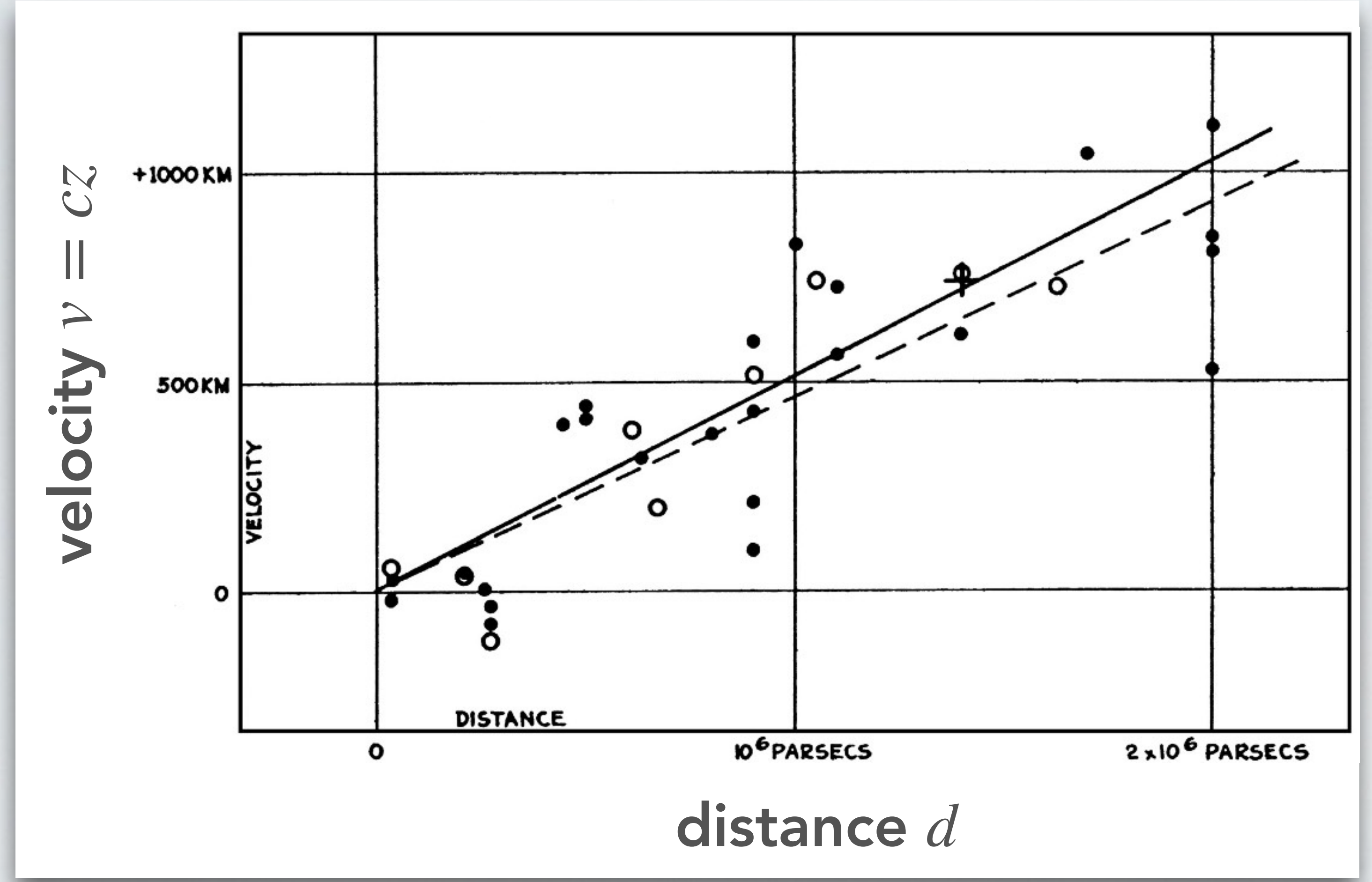
30 seconds

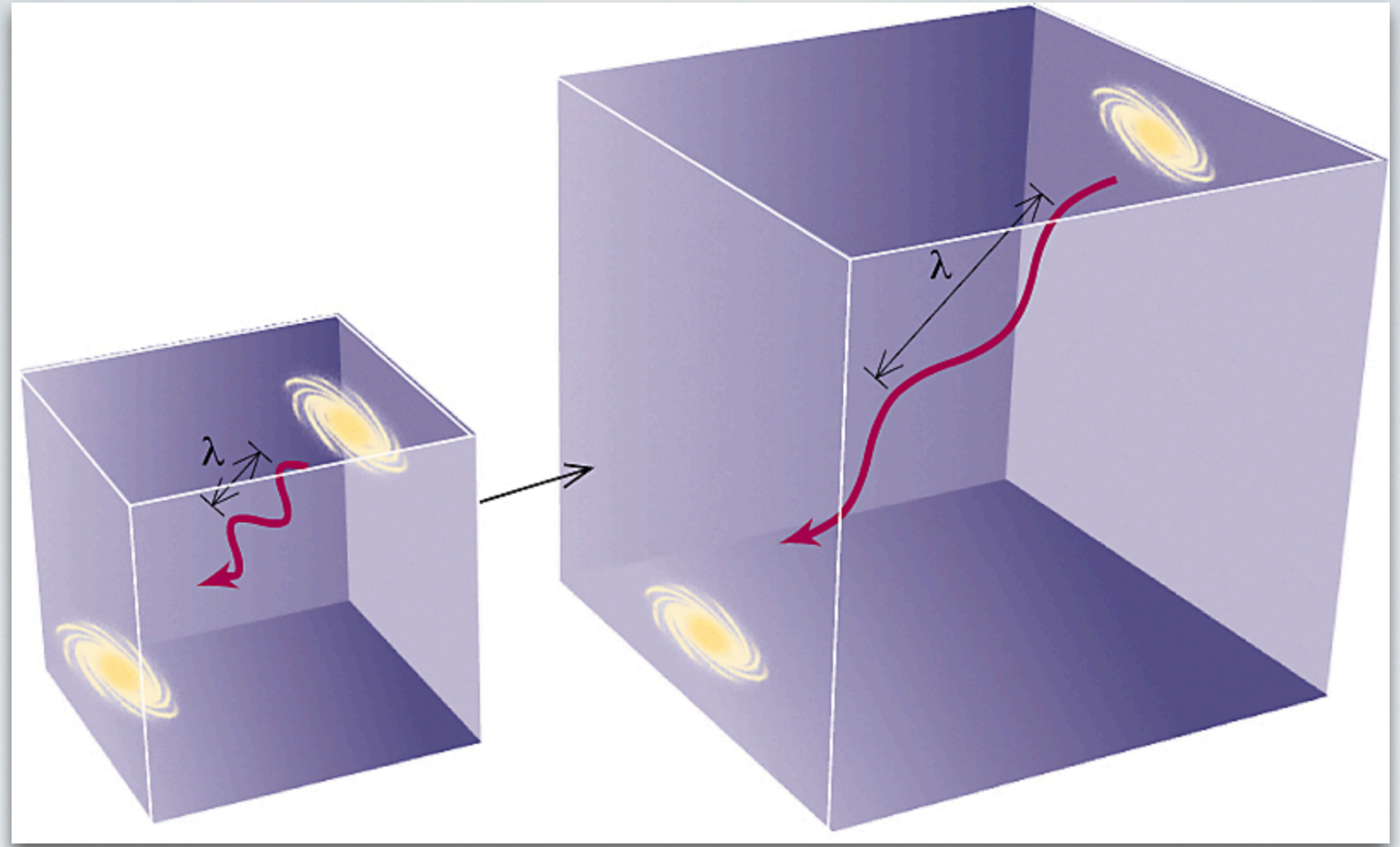
# Hubble-Lemaitre law




$$v = H_0 \times d$$

$$H_0 \approx 70 \ \frac{\text{km/s}}{\text{Mpc}}$$

# Cosmological redshift




**Redshift is caused by the expansion of space!**

# Today

- Redshift and the Size of the Universe
- The Age of the Universe (a guess)
- The Geometry of the Universe

# Part 1: Redshift and the Size of the Universe

# Metric for expanding space

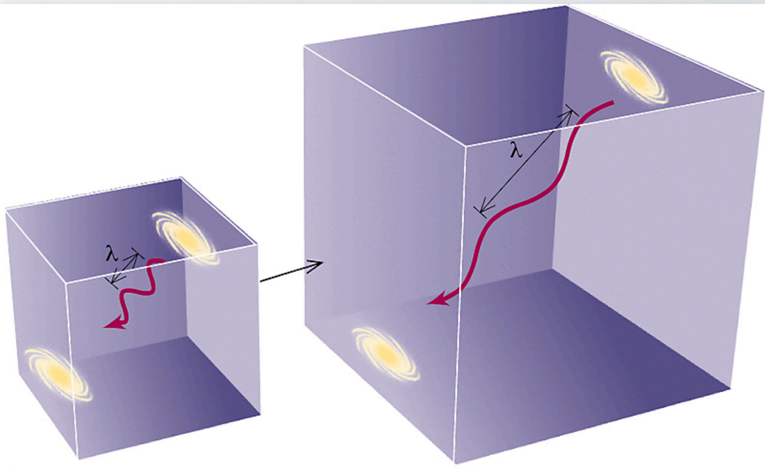
- Recall spacetime interval in flat space:

$$\Delta s_{\text{static}} = \sqrt{(c\Delta t)^2 - (\Delta x^2 + \Delta y^2 + \Delta z^2)}$$

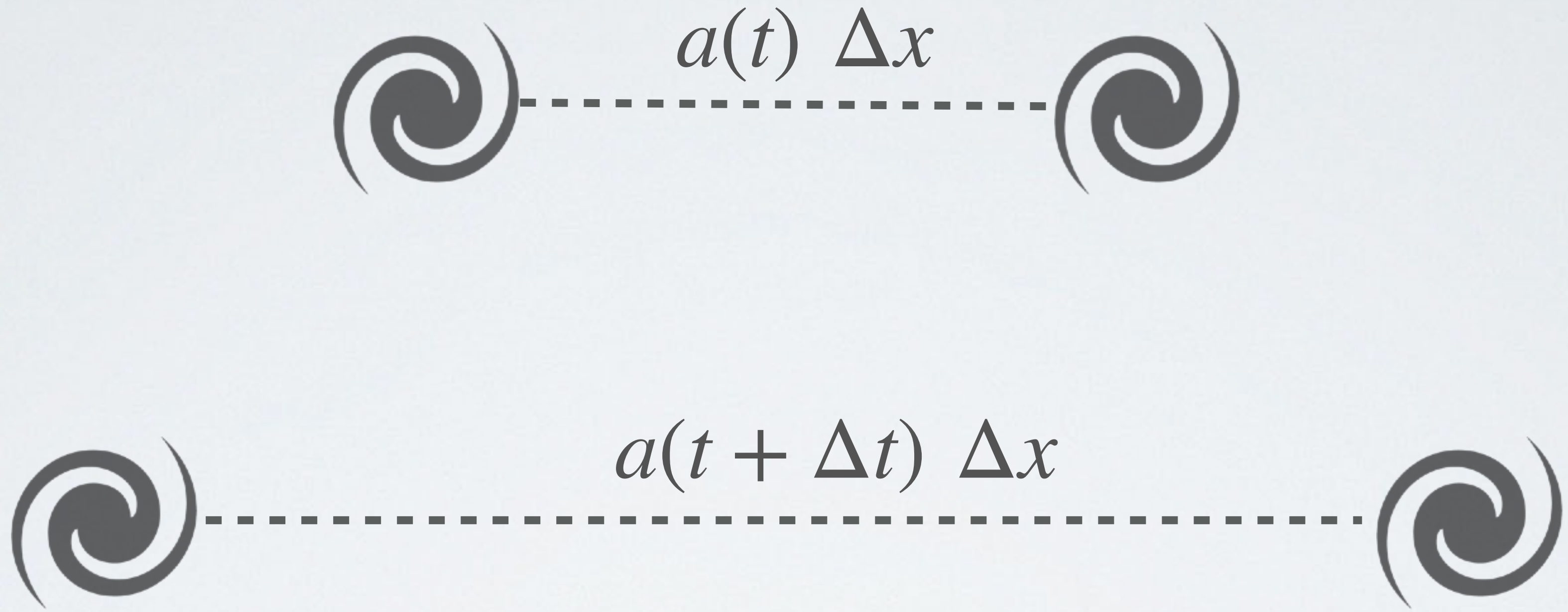
- Keep time the same, **let space expand:**

$$\Delta s_{\text{expanding}} = \sqrt{(c\Delta t)^2 - a^2(t)\ (\Delta x^2 + \Delta y^2 + \Delta z^2)}$$

- Called the (flat) Friedmann-Lemaitre-Robertson-Walker, or **FLRW metric**
- **a(t)** is the time-varying **scale factor**
- Scale factor is **unitless!**

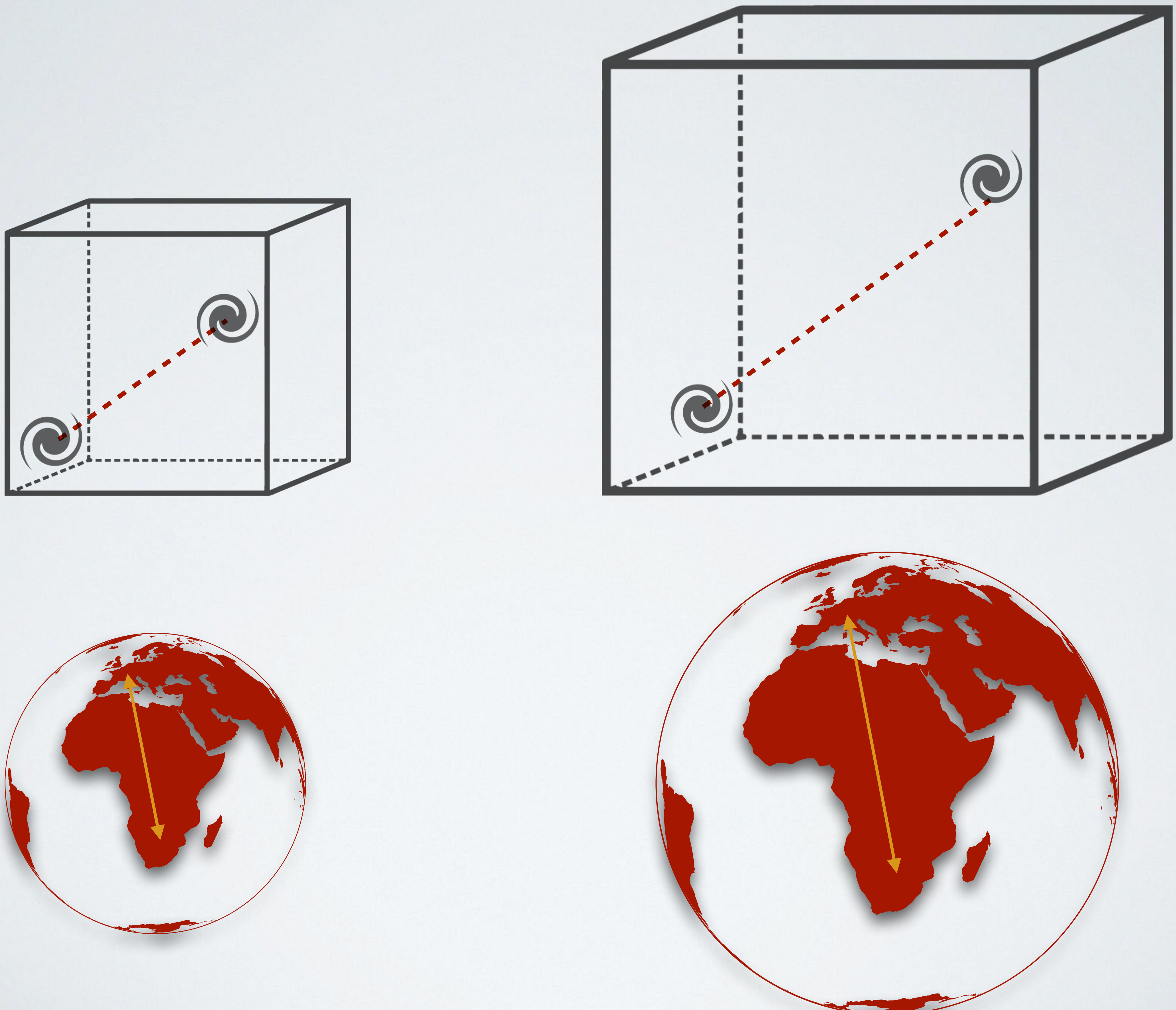


# Metric for expanding space

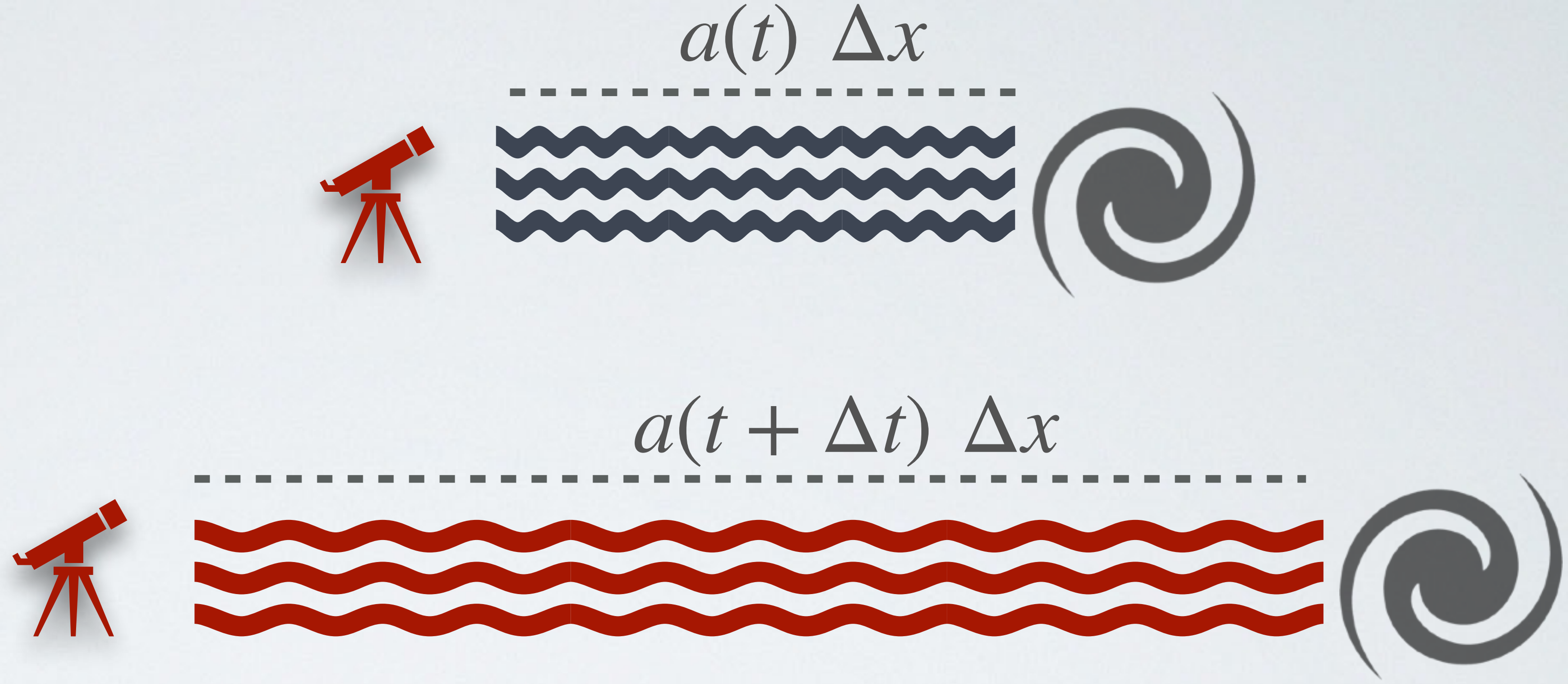



$$\Delta s_{\text{expanding}} = \sqrt{(c\Delta t)^2 - a^2(t)\ (\Delta x^2 + \Delta y^2 + \Delta z^2)}$$

# Expanding space



# Connecting redshift and scale factor




$$\lambda_{\rm obs} = \frac{a_{\rm obs}}{a_{\rm em}} \lambda_{\rm em}$$

- Redshift is not due to velocity of galaxies
  - Galaxies are **approximately stationary in space**
  - Galaxies get further apart because the **space** between them is **physically expanding**
- Cosmological redshift is due to cosmological **expansion of wavelength of light**, not the regular Doppler shift from local motions

# Connecting redshift and scale factor

**Definition of redshift:** $$z = \frac{\lambda_{\rm obs} - \lambda_{\rm em}}{\lambda_{\rm em}}$$

**Cosmological expansion:** $$\lambda_{\rm obs} = \frac{a_{\rm obs}}{a_{\rm em}} \lambda_{\rm em}$$

$$\implies z = \frac{a_{\rm obs}}{a_{\rm em}} - 1$$

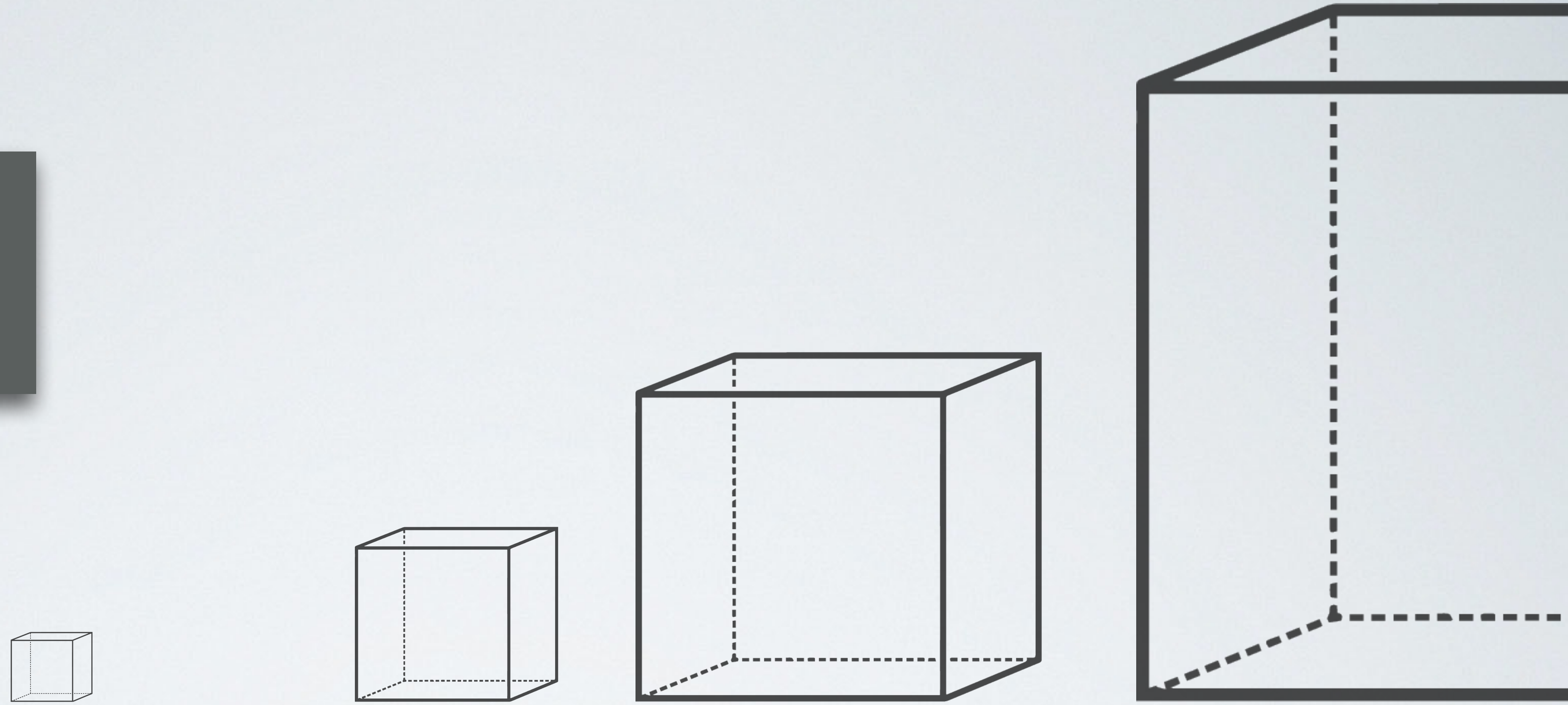
**Define scale of Universe today:** $$R_{\rm obs} = R_0 = R(t_0) = R(\text{today})$$

**Define a relative to today:** $$a(t) = R(t)/R(t_0)$$

$$\implies z = \frac{1}{a} - 1 \implies a = \frac{1}{1+z}$$

# Connecting redshift and scale factor

$$z = \frac{1}{a} - 1$$



| | | | | |
|---|---|---|---|---|
| $a = 0$ | $a = 0.2$ | $a = 0.5$ | $a = 1$ | $a = 2$ |
| $z = \infty$ | $z = 4$ | $z = 1$ | $z = 0$ | $z = -0.5$ |

Big Bang ← today → future

# Participation: Redshift-scale factor relation #1

**TurningPoint:**
What redshift does a scale factor of 0.25 correspond to?

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$$z = \frac{1}{a} - 1$$

$$a = \frac{1}{1+z}$$

40 seconds

# Participation: Redshift-scale factor relation #2

**TurningPoint:**
What scale factor does a redshift of 2 correspond to?

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$$z = \frac{1}{a} - 1$$

$$a = \frac{1}{1+z}$$

40 seconds

# Part 2: The Age of the Universe (a guess)

# Connection to Hubble rate

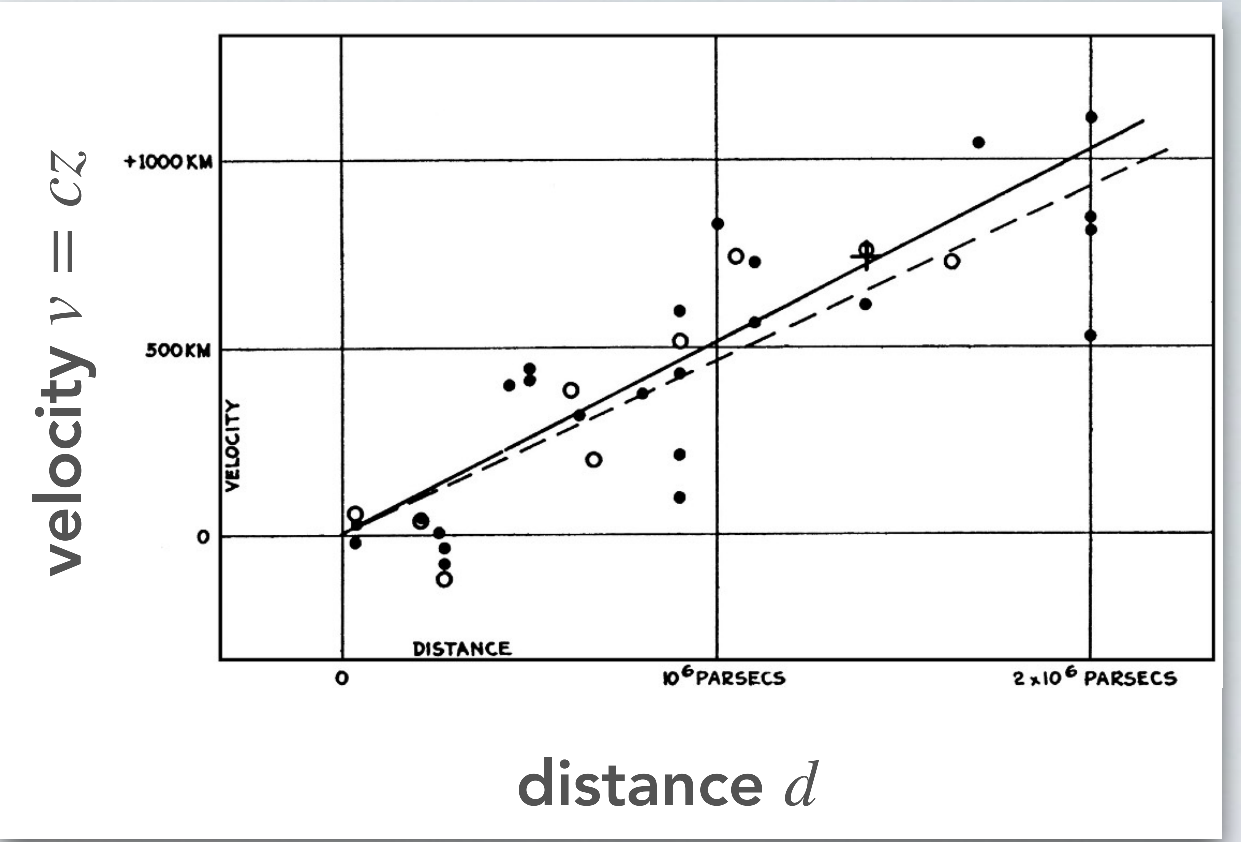
$$z = \frac{a_{\rm obs}}{a_{\rm em}} - 1 = \frac{a_{\rm obs} - a_{\rm em}}{a_{\rm em}} = \frac{\Delta a}{a}$$

For a small z, $z \approx v/c$:

$$v_{\rm Hubble} = cz = c\frac{\Delta a}{a} = c\Delta t \frac{\Delta a}{\Delta t\ a} = d \times H_0$$

This is the Hubble law if we associate

$$H(t) = \frac{1}{a(t)} \frac{da(t)}{dt}$$




$$v = H_0 \times d$$

- The **Hubble constant** is $H_0 = H(t_0)$, the **expansion rate today**
- The Hubble rate is the **fractional rate** at which the Universe expands **per time!**
  - e.g., H = 0.1/yr would mean 10% expansion per year
  - Like interest: a bank doesn't give you $10 / year, but $1 / $100 / year = 0.01 / year

# Participation: Hubble rate

**TurningPoint:**

Imagine at a = 0.5 the Universe expands to a = 0.6 in a span of one year. What is the Hubble rate?

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$$H(t) = \frac{1}{a(t)} \frac{da(t)}{dt}$$

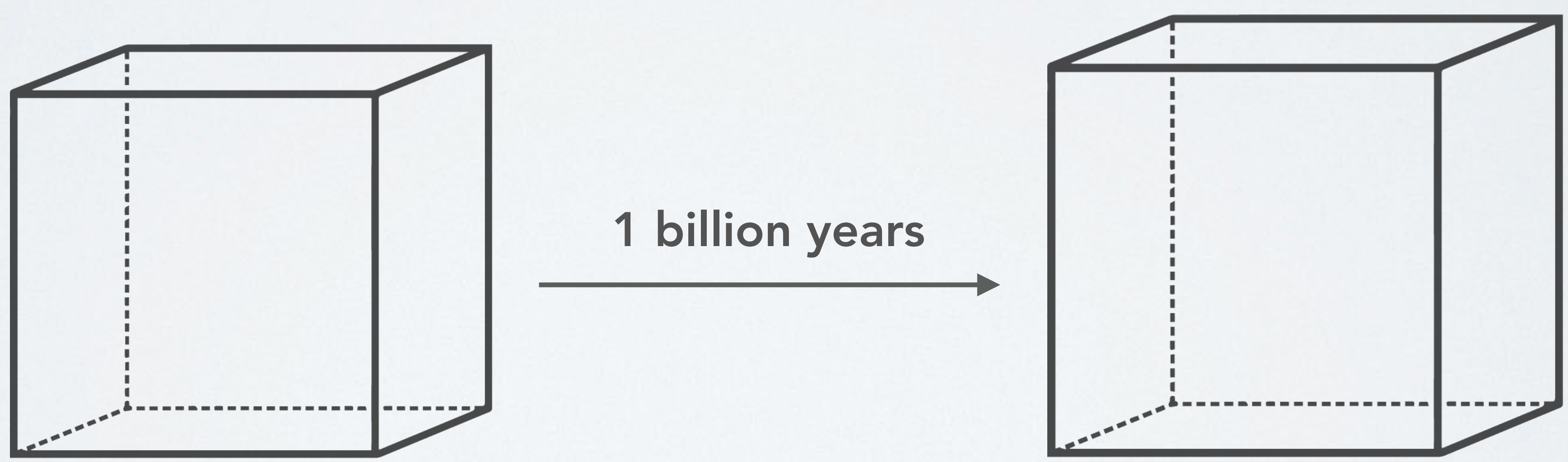
40 seconds

# How fast is the Universe expanding?

$$H_0 \approx 70\ \frac{\mathrm{km/s}}{\mathrm{Mpc}} \approx \frac{70\ \mathrm{km}}{3.1 \times 10^{19}\ \mathrm{km}} \frac{1}{\mathrm{s}} \approx \frac{2.3 \times 10^{-18}}{\mathrm{s}}$$

$$\approx \frac{7.2 \times 10^{-11}}{\mathrm{yr}}$$

$$\approx \frac{0.07}{\mathrm{Gyr}}$$




$a = 1$ $\qquad$ $a = 1.07$

# How old is the Universe?

- Imagine a Universe that is expanding at a constant rate
- $a(t) = t \times da/dt$

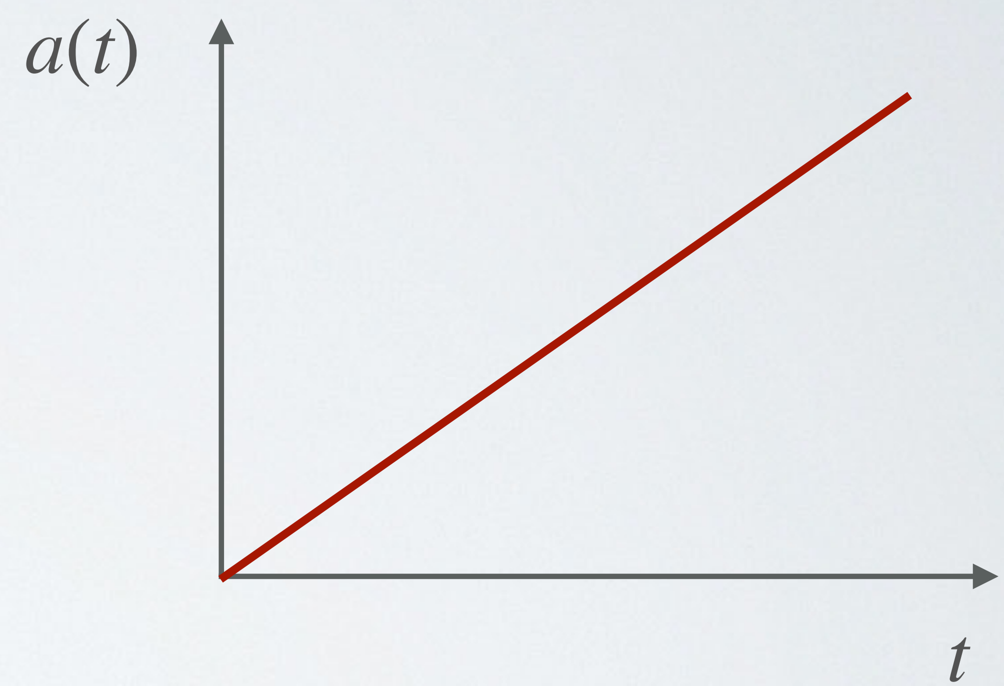
$$\implies H(t) = \frac{1}{t}$$

- With this constant expansion, the **relative rate** of expansion **decreases**
- We define the Hubble time, the time it would take to get to a certain expansion rate H if the rate is constant:

$$t_{\rm H} = \frac{1}{H(t)}$$

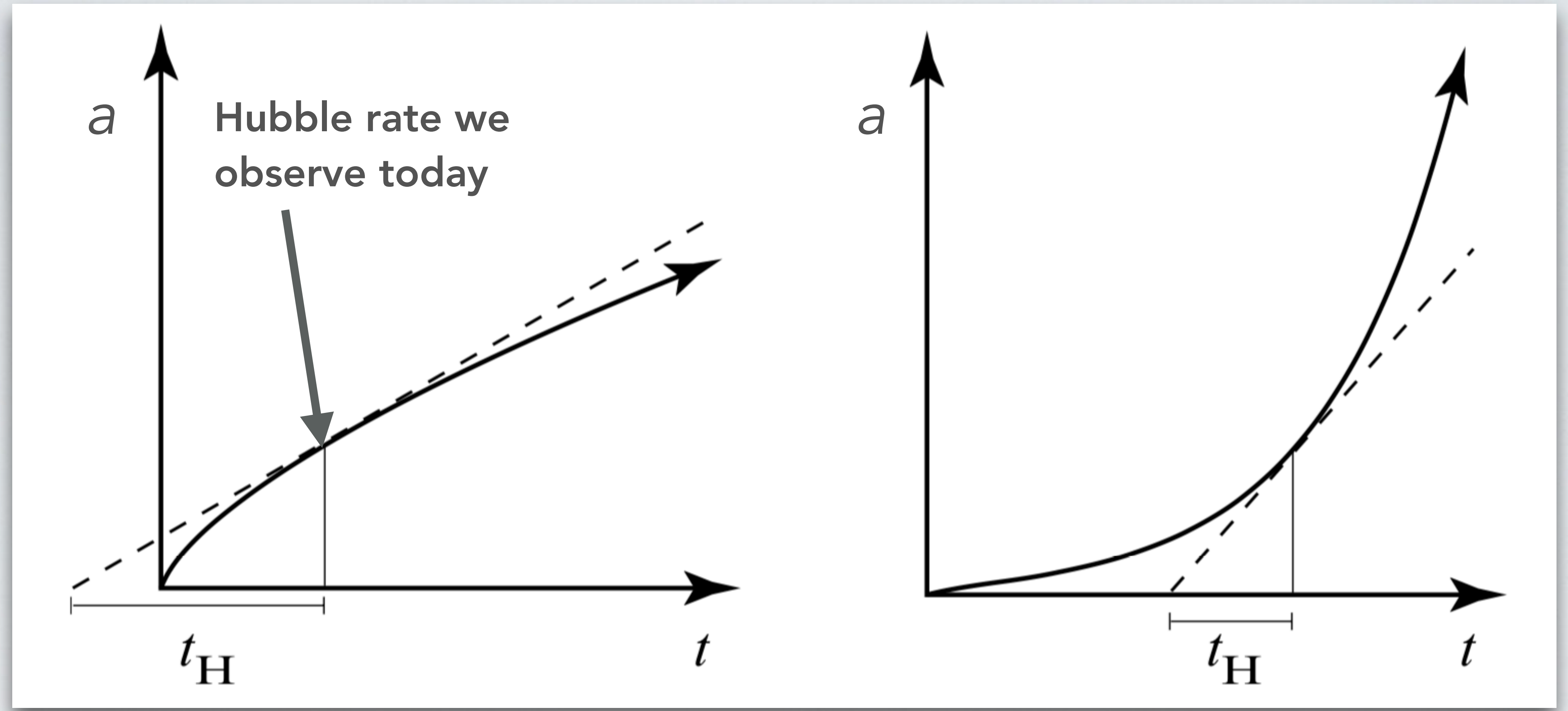
- What is this time today?
- If the Universe had been expanding at the same rate for its entire life, it would be about 13.8 Gyr old

$$H(t) = \frac{1}{a(t)}\frac{da(t)}{dt}$$




$$t_{\rm H,0} = \frac{1}{H_0} \approx \frac{1}{0.07/{\rm Gyr}} \approx 13.8\ {\rm Gyr}$$

# How old is the Universe?




Hawley & Holcomb

# Participation: Hubble time

**TurningPoint:**

Imagine we observe the distance to some galaxy to be 1 Mpc one year, and 1.2 Mpc the next year. How old would we think the Universe is, assuming the expansion rate is constant?

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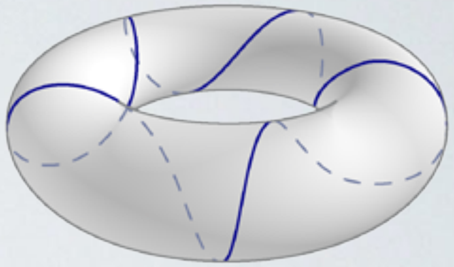
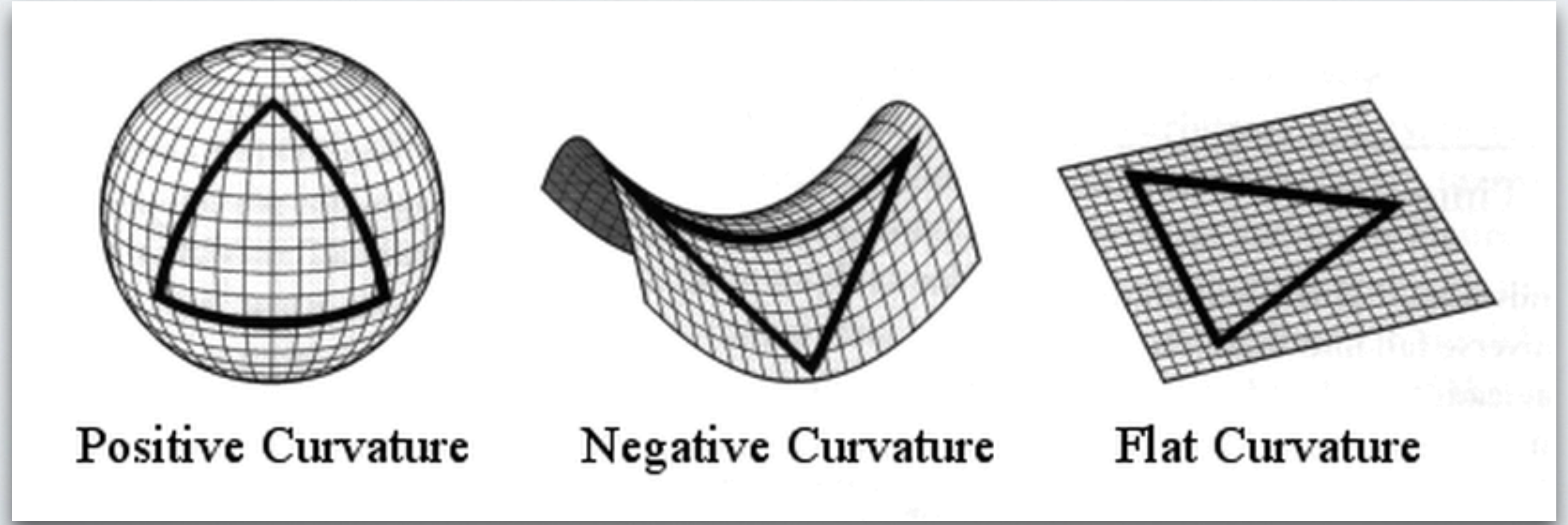
60 seconds

# Part 3: The Geometry of the Universe

# Riemann spacetimes

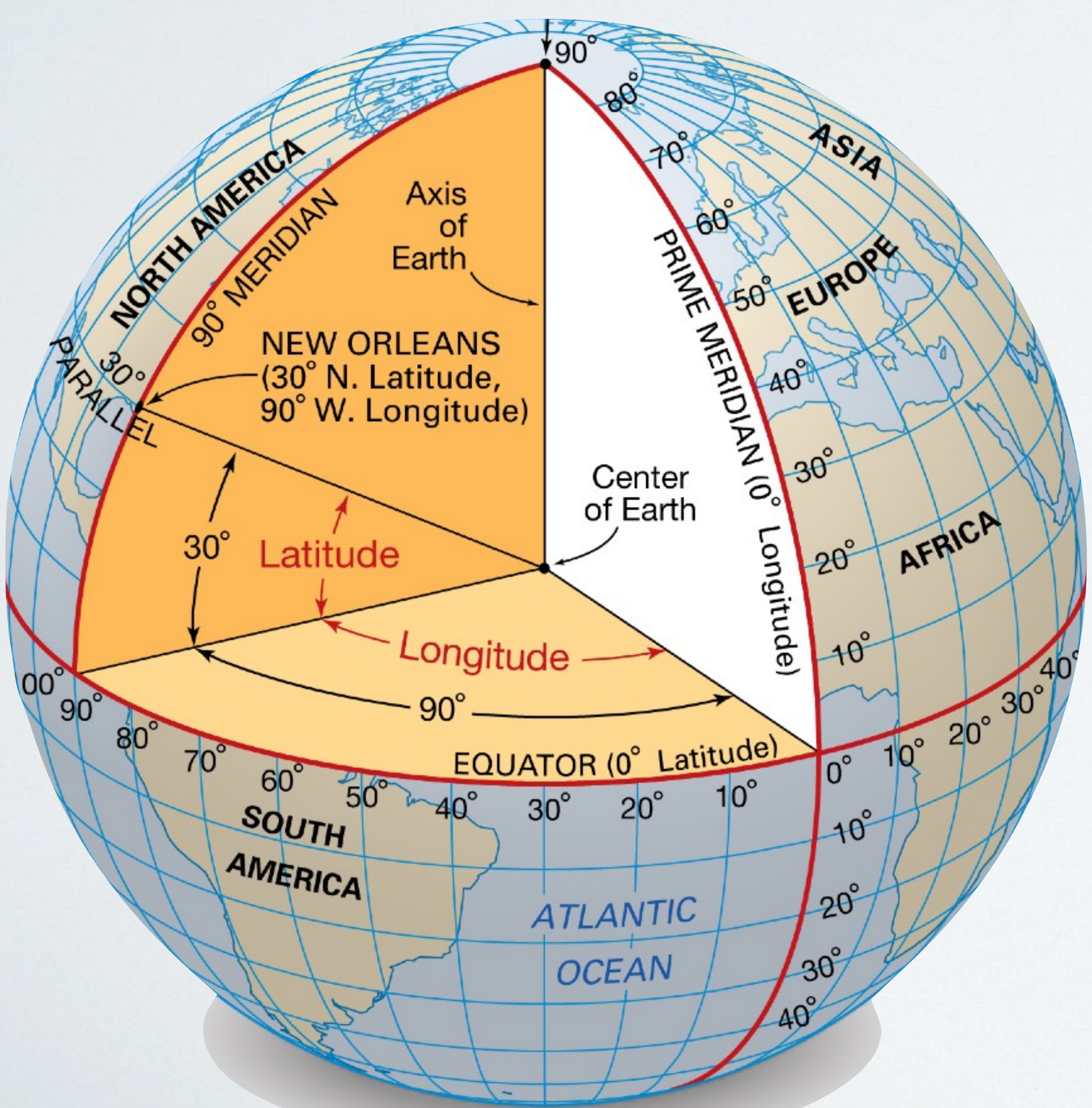
- Spacetime must be **locally flat** for strong equivalence principle
  - This is true for Riemannian spacetimes (no powers greater than 2 in metric)
  - Basically "smooth" surfaces
- For the Universe as a whole, must be **homogeneous & isotropic**
  - **Flat** (Euclidean space, Minkowski spacetime)
  - **Positively curved** (like a sphere)
  - **Negatively curved** or **hyperbolic** (like a saddle point everywhere; but no equivalent in 2D/3D)

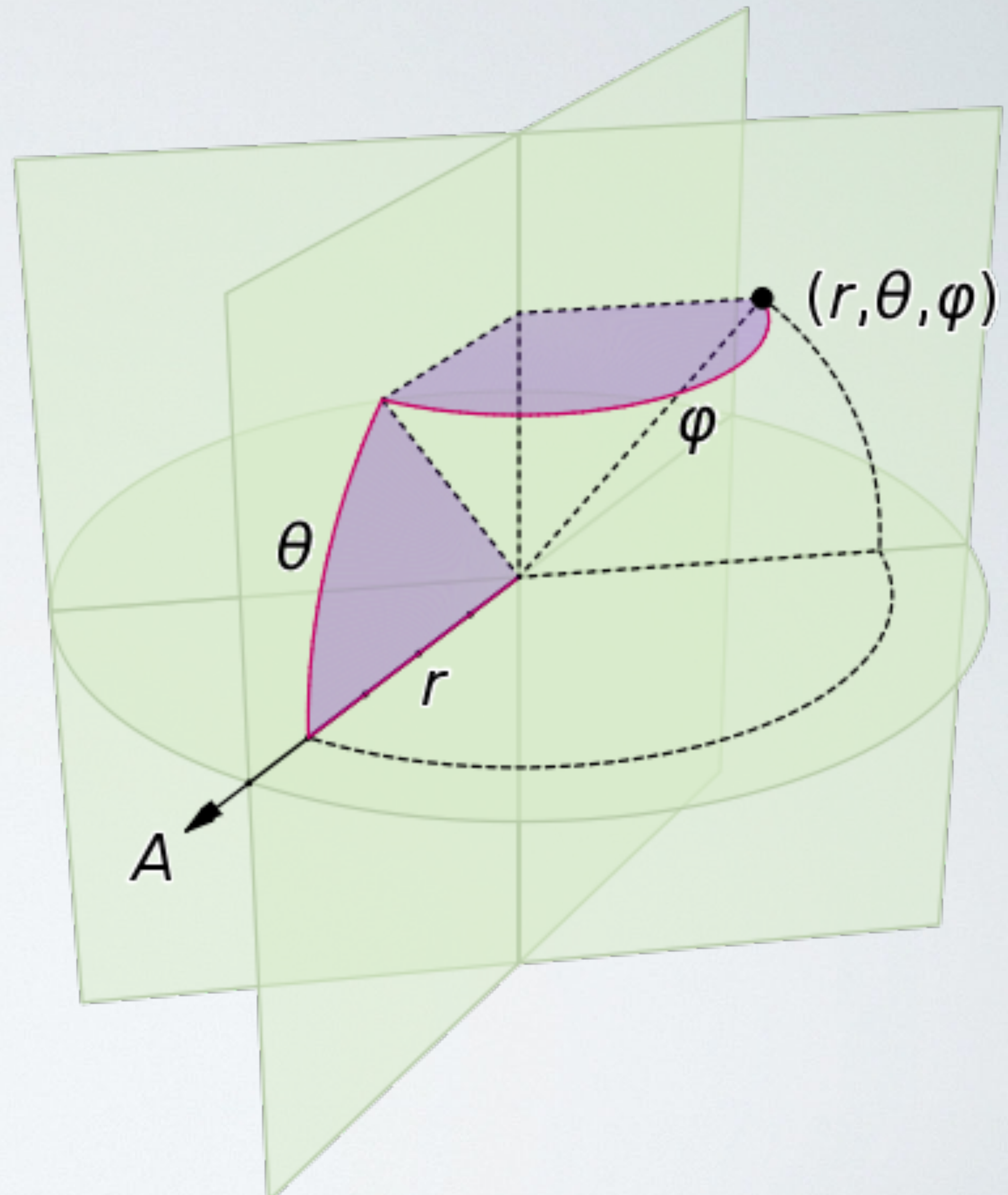




# Spherical coordinates

- Spherical coordinates:
  - Radius ($r$)
  - Angle "up-down" ($\theta$)
  - Angle "around" ($\phi$)




Images: Encyclopaedia Brittanica

# Metric for curved and expanding space

$$\Delta s_{\text{FLRW,flat}} = \sqrt{(c\Delta t)^2 - a^2(t)\ (\Delta x^2 + \Delta y^2 + \Delta z^2)}$$

$$\Longrightarrow \quad \Delta s_{\text{FLRW,flat}} = \sqrt{(c\Delta t)^2 - a^2(t)\ \ [\Delta r^2 + r^2(\Delta\theta^2 + \sin^2(\theta)\Delta\phi^2)]}$$

$$\Delta s_{\text{FLRW,curved}} = \sqrt{(c\Delta t)^2 - a^2(t)\ \ \left[\frac{\Delta r^2}{1 - kr^2} + r^2(\Delta\theta^2 + \sin^2(\theta)\Delta\phi^2)\right]}$$

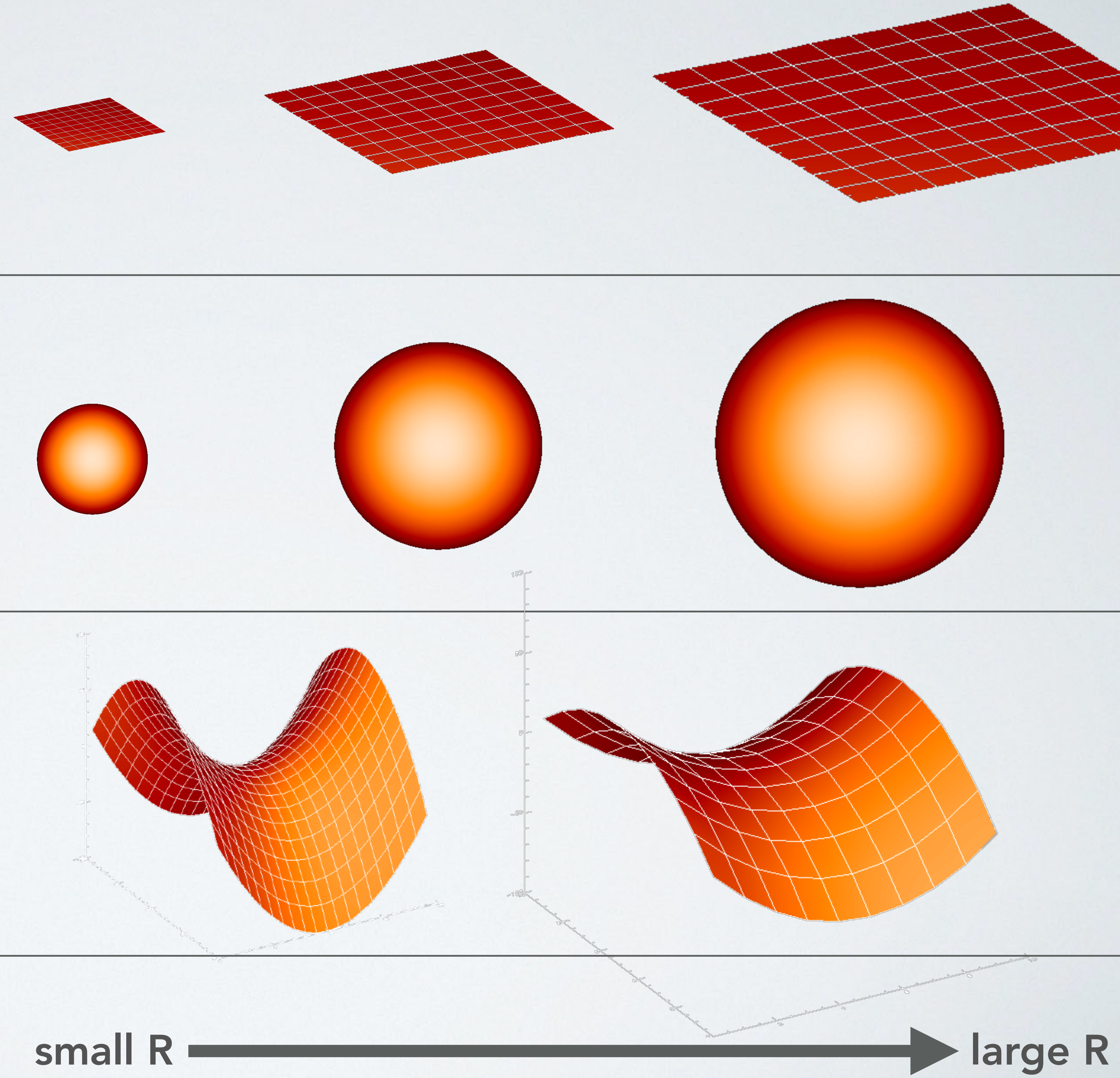
**k can be 0 (flat), >0 (positively curved), <0 (negatively curved)**

# Geometry of space

k = 0
Flat space

k > 0 ($k = +1/R^2$)
Positively curved

k < 0 ($k = -1/R^2$)
Negatively curved
(hyperbolic)



# Take-aways

- **Cosmological redshift** is caused by the expansion of space
- The **scale factor a(t)** measures the size of the Universe relative to today; it is related to redshift as $a = 1 / (1 + z)$
- The **Hubble rate H(t)** measures the relative expansion of the Universe per time
- The **FLRW metric** describes expanding space, which can be **flat or positively/negatively curved**

# Next time…

**We'll talk about:**

- Dynamics of the Universe

**Assignments**

- Post-lecture quiz (by tomorrow night)
- Homework #2 (due tonight!)

**Reading:**

- H&H Chapter 11